

Another Example of Logarithmic Differentiation

This example could be done equally well by converting to base e , but we're going to do it using logarithmic differentiation. Recall that the rule we use for logarithmic differentiation is $(\ln u)' = u'/u$.

Here we have a “moving” (non-constant) exponent and a moving base.

Example: Let $v = x^x$. Find v' .

First, we take the natural log of both sides to see that $\ln v = \ln(x^x) = x \ln x$.

Next, we differentiate both sides of the equation, using the product rule and the rule for the derivative of $\ln x$ on the right hand side:

$$(\ln v)' = \ln x + x \cdot \frac{1}{x}.$$

Now apply the formula $(\ln u)' = u'/u$. to get:

$$v'/v = 1 + \ln x$$

Plugging in x^x for v and solving for v' , we get:

$$\begin{aligned}\frac{v'}{x^x} &= 1 + \ln x \\ v' &= x^x(1 + \ln x) \\ \frac{d}{dx}x^x &= x^x(1 + \ln x)\end{aligned}$$

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