Implicit Differentiation and the Chain Rule

The chain rule tells us that:

$$\frac{d}{dx}(f \circ g) = \frac{df}{dg}\frac{dg}{dx}.$$

While implicitly differentiating an expression like $x + y^2$ we use the chain rule as follows:

$$\frac{d}{dx}(y^2) = \frac{d(y^2)}{dy}\frac{dy}{dx} = 2yy'.$$

Why can we treat y as a function of x in this way?

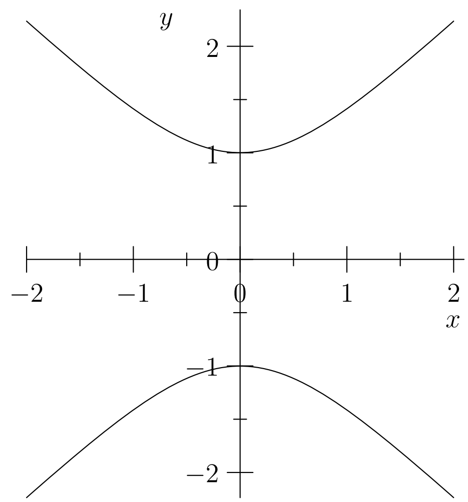

Figure 1: The hyperbola $y^2 - x^2 = 1$.

Consider the equation $y^2 - x^2 = 1$, which describes the hyperbola shown in Figure 1. We cannot write y as a function of x, but if we start with a point (x, y) on the graph and then change its x coordinate by sliding the point along the graph its y coordinate will be constrained to change as well. The change in y is *implied* by the change in x and the constraint $y^2 - x^2 = 1$. Thus, it makes sense to think about $y' = \frac{dy}{dx}$, the rate of change of y with respect to x.

Given that $y^2 - x^2 = 1$:

a) Use implicit differentiation to find y'.

b) Check your work by using Figure 1 to estimate the slope of the tangent line to the hyperbola when $y = -1$ and when $x = 1$.

c) Check your work for $y > 0$ by solving for y and using the direct method to take the derivative.

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