Slope as Ratio

While we're still thinking geometrically, we can now use symbols and formulas in our computation.

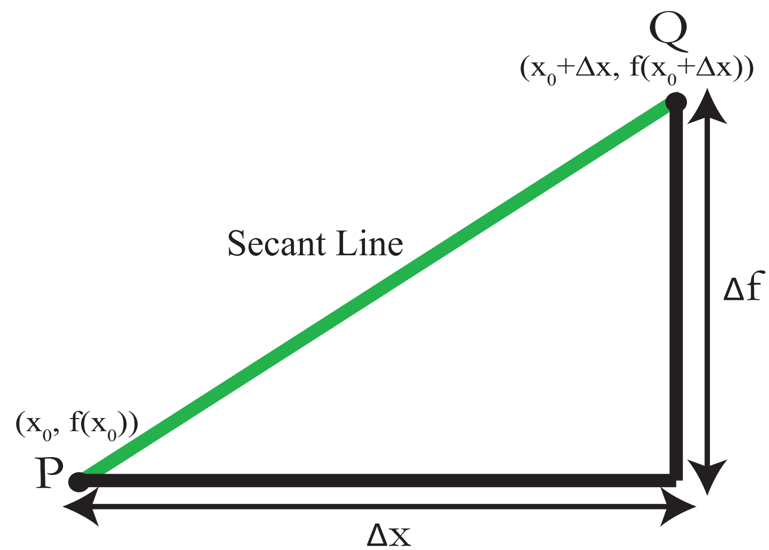

Figure 1: Geometric definition of the derivative

We start with a point $P = (x_0, f(x_0))$. We move over a tiny horizontal distance Δx (pronounced "delta x" and also called "the change in x") and find point $Q = (x_0 + \Delta x, f(x_0 + \Delta x))$. These two points lie on a secant line of the graph of $f(x)$; we will compute the slope of this line. The vertical difference between P and Q is $\Delta f = f(x_0 + \Delta x) - f(x_0)$.

The slope of the secant PQ is rise divided by run, or the ratio $\frac{\Delta f}{\Delta x}$. We've said that the tangent line is the limit of the secant lines. It is also true that the slope of the tangent line is the limit of the slopes of the secant lines. In other words,

$$m = \lim_{Q->P} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}.$$