18.01 Single Variable Calculus
Fall 2006

18.01 Practice Questions for Exam 1

Solutions will be posted on the 18.01 website.

No books, notes, or calculators will be allowed at the exam.

1. Evaluate each of the following, simplifying where possible; for (b) indicate reasoning. The letters a and k represent constants.

$$a)\quad \frac{d}{dt}\left(\frac{3t}{\ln t}\right)\bigg|_{e^2} \qquad b)\quad \lim_{u\to 0}\frac{3u}{\tan 2u} \qquad c)\quad \frac{d^3}{dx^3}\sin kx \qquad d)\quad \frac{d}{d\theta}\sqrt[3]{a+k\sin^2\theta}$$

2. Derive the formula for $\frac{d}{dx}x^3$ at the point $x = x_0$ directly from the definition of derivative.

3. Find $\lim\limits_{h\to 0}\frac{1-\sqrt[3]{1+h}}{h}$ by relating it to a derivative. (Indicate reasoning.)

4. Sketch the curve $y = \sin^{-1} x$, $-1 \le x \le 1$, and derive the formula for its derivative from that for the derivative of $\sin x$.

5. For the function

$$f(x) = \begin{cases} ax+b, & x > 0 \\ 1-x+x^2, & x \le 0, \end{cases}, \qquad a \text{ and } b \text{ constants},$$

a) find all values of a and b for which the function will be continuous;

b) find all values of a and b for which the function will be differentiable.

6. For the curve given by the equation

$$x^2y + y^3 + x^2 = 8,$$

find all points on the curve where its tangent line is horizontal.

7. Where does the tangent line to the graph of $y = f(x)$ at the point (x_0, y_0) intersect the x-axis?

8. The volume of a spherical balloon is decreasing at the instantaneous rate of -10 cm^3/sec, at the moment when its radius is 20 cm. At that moment, how rapidly is its radius decreasing?

9. Where are the following functions discontinuous?

$$a)\quad \sec x \qquad\qquad b)\quad \frac{1+x^2}{1-x^2} \qquad\qquad c)\quad \frac{d}{dx}\,|\,x\,|$$

10. A radioactive substance decays according to a law $A = A_0e^{-rt}$, where $A(t)$ is the amount in present at time t, and r is a positive constant.

a) Derive an expression in terms of r for the time it takes for the amount to fall to one-quarter of the initial amount A_0.

b) At the moment when the amount has fallen to 1/4 the initial amount, how rapidly is the amount falling? (Units: grams, seconds.)