Lecture 2. September 9, 2005

Homework. Problem Set 1 Part I: (f)–(h); Part II: Problems 3.

Practice Problems. Course Reader: 1C-2, 1C-3, 1C-4, 1D-3, 1D-5.

1. Tangent lines to graphs. For $y = f(x)$, the equation of the *secant line* through $(x_0, f(x_0))$ and $(x_0 + \Delta x, f(x_0 + \Delta x))$ is,

$$y = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}(x - x_0) + f(x_0).$$

In the limit, the equation of the *tangent line* through $(x_0, f(x_0))$ is,

$$y = f'(x_0)(x - x_0) + y_0.$$

Example. For the parabola $y = x^2$, the derivative is,

$$y'(x_0) = \boxed{2x_0.}$$

The equation of the tangent line is,

$$y = 2x_0(x - x_0) = \boxed{2x_0 x - x_0^2.}$$

For instance, the equation of the tangent line through $(2, 4)$ is,

$$y = \boxed{4x - 4.}$$

Given a point (x, y), what are all points (x_0, x_0^2) on the parabola whose tangent line contains (x, y)? To solve, consider x and y as constants and solve for x_0. For instance, if $(x, y) = (1, -3)$, this gives,

$$(-3) = 2x_0(1) - x_0^2,$$

or,

$$x_0^2 - 2x_0 - 3 = 0.$$

Factoring $(x_0-3)(x_0+1)$, the solutions are x_0 equals -1 and x_0 equals 3. The corresponding tangent lines are,

$$y = -2x - 1,$$

and

$$y = 6x - 9.$$

For general (x, y), the solutions are,

$$x_0 = \boxed{x \pm \sqrt{x^2 - y}.}$$

2. Limits. Precise definition is on p. 791 of Appendix A.2. Intuitive definition: $\lim_{x\to x_0} f(x)$ equals L if and only if all values of $f(x)$ can be made arbitrarily close to L by choosing x sufficiently close to x_0. One interpretation is the "microscope/laser illuminator" analogy: An observer focuses a microscopes field-of-view on a thin strip parallel to the x-axis centered on $y = L$. The goal of the illuminator is to focus a laser-beam centered on x_0 parallel to the y-axis (but with the line $x = x_0$ deleted) so that only the portion of the graph in the field-of-view is illuminated. If for every magnification of the microscope, the illuminator can succeed, then the limit is defined and equals L.

There is a beautiful Java applet on the webpage of Daniel J. Heath of Pacific Lutheran University,

`http://www.plu.edu/~heathdj/java/calc1/Epsilon.html`

If you use this, try $a = -1$.

For left-hand limits, use a laser that illuminates only to the left of x_0. For right-hand limits, use a laser that illuminates only to the right of x_0.

3. Continuity. A function $f(x)$ is **continuous** at x_0 if $f(x_0)$ is defined, $\lim_{x\to x_0} f(x)$ is defined, and $\lim_{x\to x_0} f(x)$ equals $f(x_0)$. Also, $f(x)$ is continuous on an interval if it is continuous at every point of the interval. The types of discontinuity are: removable discontinuity, jump discontinuity, infinite discontinuity and essential discontinuity.