18.01 Exam 1

Name: _____

Problem 1:	_____	/15
Problem 2:	_____	/10
Problem 3:	_____	/15
Problem 4:	_____	/20
Problem 5:	_____	/25
Problem 6:	_____	/15
Total:	_____	/100

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Name: _____ **Problem 1**: _____ /15

Problem 1(15 points) Use the definition of the derivative as a limit of difference quotients to compute the derivative of $y = x + \frac{1}{x}$ for all points $x > 0$. Show all work.

Name: _____ **Problem 2**: _____ /10

Problem 2(10 points) For the function $f(x) = e^{-x^2/2}$, compute the first, second and third derivatives of $f(x)$.

Extra credit(5 points) Only attempt this after you have completed the rest of the exam and checked your answers. For every positive integer n, show that the n^{th} derivative of $f(x)$ is of the form $f^{(n)}(x) = p_n(x)f(x)$, where $p_n(x)$ is a polynomial. Also, give a rule to compute $p_{n+1}(x)$, given $p_n(x)$.

Name: _____ **Problem 3**: _____ /15

Problem 3(15 points) A function $y = f(x)$ satisfies the implicit equation,

$$2x^3 - 9xy + 2y^3 = 0.$$

The graph contains the point $(1, 2)$. Find the equation of the tangent line to the graph of $y = f(x)$ at $(1, 2)$.

Name: _____ **Problem 4**: _____ /20

Problem 4(20 points) The point $(0, 4)$ is **not** on the graph of $y = x + 1/x$, but it is contained in exactly one *tangent line* to the graph.

(a)(15 points) Find the one value of a for which the tangent line to the graph of $y = x + 1/x$ at $(a, a + 1/a)$ contains $(0, 4)$.
Hint: You do *not* need to solve a quadratic equation to find a.

(b)(5 points) Write the equation of the corresponding tangent line.

Name: _____ **Problem 5**: _____ /25

Problem 5(25 points) In an automobile crash-test, a car is accelerated from rest at 2 m/s^2 for 5 seconds and then decelerated at $-4m/s^2$ until it strikes a barrier. The position function is,

$$s(t) = \begin{cases} t^2 & 0 \leq t < 5 \\ -2t^2 + At + B & t \geq 5 \end{cases}$$

(a)(10 points) Assuming that both $s(t)$ and $s'(t)$ are continuous at $t = 5$, determine A and B.

(b)(15 points) The barrier is located at $s = 33$ meters. Determine the velocity of the car when it strikes the barrier. (The quadratic polynomial has whole number roots.)

Name: _____ **Problem 6**: _____ /15

Problem 6(15 points) For each of the following functions, compute the derivative. Show all work.

(a)(4 points) $y = (e^x - e^{-x})/(e^x + e^{-x})$

(b)(3 points) $y = x\ln(x) - x$

(c)(3 points) $y = \sqrt{1 + x^{1234}}$

(d)(5 points) $y = \log_{10}(x^3 + 3x)$.