18.01 EXAM 5
NOVEMBER 20, 2003

Name: ______________________________________

Problem 1:	__________	/20
Problem 2:	__________	/15
Problem 3:	__________	/25
Problem 4:	__________	/20
Problem 5:	__________	/20
Total:	__________	/100

Please write the hour of your recitation.

Hour: ____________

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Date: Fall 2003.

Name: ________________________________ **Problem 1**: ______________ /20

Problem 1(20 points) Compute the partial fraction decomposition of $f(z) = \frac{1}{(z^2-1)^2}$. Show all your work.

Hint: Use Heaviside's method to determine 2 of the 4 undetermined coefficients. Is $f(z)$ an even function or an odd function? What does this tell you about the 2 remaining coefficients? Combine this with a substitution to determine the 2 remaining coefficients (or use any other method you like to determine the 2 coefficients).

Name: ______________________________________ **Problem 2**: ________________ /15

Problem 2(15 points) Use polynomial division to compute the following indefinite integral (for $x > 1$).

$$\int \frac{x^3 - 1}{(x-1)^2} dx.$$

Show all your work.

Name: ____________________________________ **Problem 3**: ________________ /25

Problem 3(25 points) This problem computes a recursive relation for the antiderivative of $\sec^n(\theta)$.

(a)(10 points): Compute the derivative of $\sec^{n-2}(\theta)\tan(\theta)$. Eliminate $\tan(\theta)$ from your answer using a trigonometry identity in order to express the derivative in terms of $\sec(\theta)$ alone. Show all your work.

(b)(10 points): Write your answer from (a) in the form of an integral identity, and then solve for $\int \sec^n(\theta)d\theta$ to find a recursive formula of the form

$$\int \sec^n(\theta)d\theta = F(\sec(\theta), \tan(\theta)) + A\int \sec^{n-2}(\theta)d\theta.$$

Show all your work.

(c)(5 points): Use your recursive formula to compute the antiderivative of $\sec^4(\theta)$. Show all your work.

Name: ______________________________________ **Problem 4**: ________________ /20

Problem 4(20 points)

(a)(15 points): Use a trigonometric substitution of the form $x = f(\theta)$ to express the following indefinite integral in terms of the variable θ (assume $x > 1$).

$$\int \frac{1}{x^2\sqrt{x^2-1}}dx.$$

Show all your work.

(b)(5 points): Using trigonometry identities, back-substitute to express the indefinite integral in terms of the original variable x. Show all your work (do not simply copy a formula from your index card).

Name: ____ **Problem 5**: _________________ /20

Problem 5(20 points) In each of the following, use completing the square and a trigonometric substitution to evalute the indefinite integral.

(a)(10 points) $\int \frac{1}{\sqrt{5x-x^2}}dx$ (assume that $5x - x^2 > 0$). Show all your work.

(b)(10 points) $\int \frac{1}{x^2-6x+18}dx$. Show all your work.