Prob. 19.6 - storage and loss moduli vs. frequency, SLS model

Debye eqns. for storage and loss moduli:

```
> E_stor:=k[e]+k[1]*omega^2*tau^2/(1+omega^2*tau^2);
```

$$E\_stor := k_e + \frac{k_1\,\omega^2\,\tau^2}{1+\omega^2\,\tau^2}$$

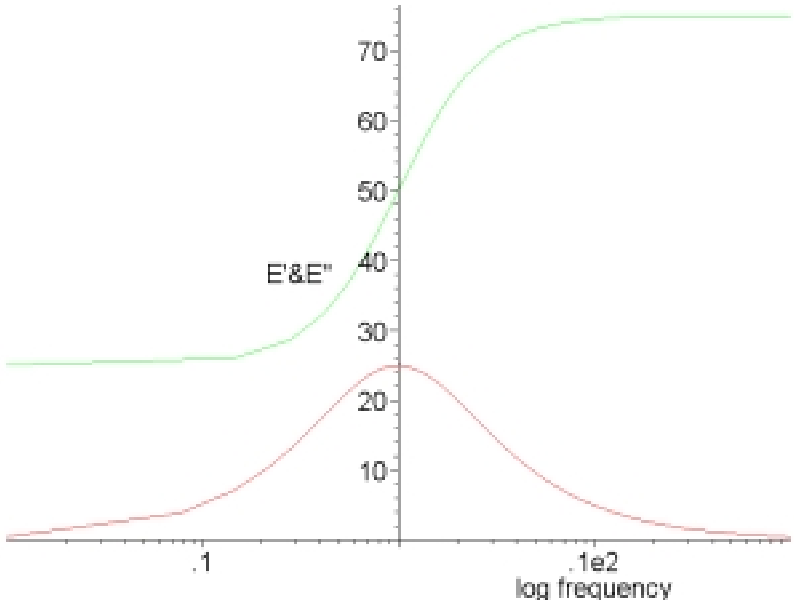
```
> E_loss:=k[1]*omega*tau/(1+omega^2*tau^2);
```

$$E\_loss := \frac{k_1\,\omega\,\tau}{1+\omega^2\,\tau^2}$$

```
> Digits:=4:k[e]:=25:k[1]:=50:tau:=1:
```

Semilog plot (E' in green, E" in red):

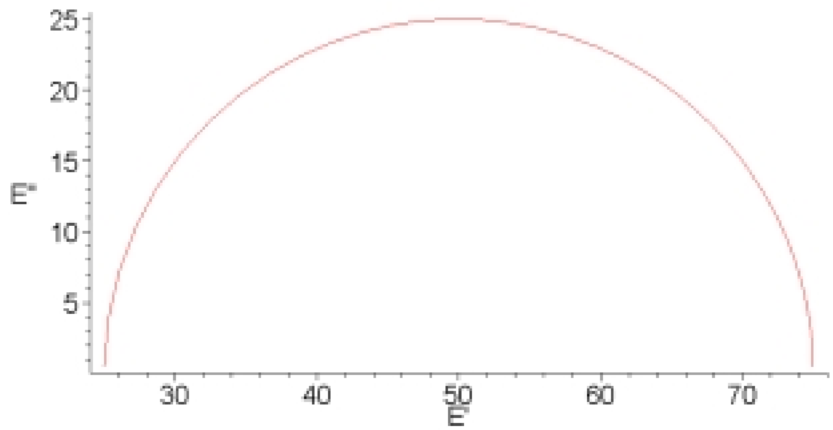
```
> with(plots):semilogplot({E_stor,E_loss},omega=.01
  ..100,labels=[`log frequency`,`E'&E"    `]);
```



Note maximum in E" and inflection in E' occurs when omega*tau =1, and maximum in E" is half the relaxation strength of E'.

Argand (Cole-Cole) plot:

```
> plot([E_stor(omega),E_loss(omega),omega=.01..100],scaling=constrai
  ned,labels=[`E'`,`E"   `]);
```



Semicircle of radius equal to half the relaxation strength of E'.

Prob. 19.7 - Voigt-SLS model compliance

Voigt arm:

$$\sigma = k_v \varepsilon_v + \eta \dot{\varepsilon}$$

Dividing by k_v, using $C_v = 1/k_v$ and $\tau = \eta / k_v$, and taking transforms::

$$\frac{1}{k_v}\bar{\sigma} = \bar{\varepsilon}_v + \frac{\eta}{k_v} s\bar{\varepsilon}_v = (1+\tau s)\bar{\varepsilon}_v \rightarrow \bar{\varepsilon}_v = \frac{C_v \bar{\sigma}}{\tau\left(s+\frac{1}{\tau}\right)}$$

Adding the glassy-response spring:

$$\bar{\varepsilon} = \bar{\varepsilon}_g + \bar{\varepsilon}_v = C_g \bar{\sigma} + \frac{C_v \bar{\sigma}}{\tau\left(s+\frac{1}{\tau}\right)}$$

Compliance operator:

$$C = \frac{\bar{\varepsilon}}{\bar{\sigma}} = C_g + \frac{C_v}{\tau\left(s+\frac{1}{\tau}\right)}$$

Prob. 19.8 - Voigt-SLS creep compliance

Compliance operator:

```
> C_op:=C[g]+C[v]/(tau*(s+1/tau));
```

$$C\_op := C_g + \frac{C_v}{\tau\left(s + \frac{1}{\tau}\right)}$$

For creep: $\bar{\varepsilon} = C\bar{\sigma} = C\frac{\sigma_o}{s} \rightarrow \frac{\bar{\varepsilon}}{\sigma_o} = \bar{C}_{crp} = \frac{C}{s}$

Divide compliance operator by s, take inverse transform, collect terms:

```
> with(inttrans):C_crp:=collect(invlaplace(C_op/s,s,t),C[v]);
```

$$C\_crp := \left(-\mathbf{e}^{\left(-\frac{t}{\tau}\right)} + 1\right) C_v + C_g$$
