PROFESSOR: Everyone seems to be here. At least all the seats are filled up, so why don't we begin. One thing that I did finally, to be specific, I told you that we would have three quizzes and that they would be uniformly spaced one third, and 2/3 and 2.89 thirds of the way through the term. And I figured out where those dates would be. So since I have them, let me give you the date.

So Quiz 1 is going to be on Thursday, October 6. Quiz 2 will be on Tuesday, November 8, and Quiz 3 will be on Thursday, December 8. And this, as advertised, is exactly one third of the way through the lectures. This is exactly 2/3 of the way through the lectures, and this is the next to the last meeting that we'll have. This is the week before the final week of the terms. so I deliberately kept it a little bit away from the usual end-of-term crunch.

Being person who is sensitive to symmetry and order, I find it highly regrettable that we cannot have the quiz on October 8. Then you would on the 8th of October, November, December, you have a quiz. Unfortunately, October 8, much as that would beautify the schedule, is on a Saturday, so we can't do it so. I could do it, but I don't think anybody would come.

I've been, believe it or not, going through these problem sets in great detail. I didn't want to hand them out until I had the list-- and you may or may not know-- that is made up for the registrants in each class, And we get a nice array of photographs with names and department numbers and years numbers under them. That something, by the way, that only the instructor in a class requests, not even the secretary can do it. So this is not widely broadcast.

I found to my dismay though when I finally got it that about one third of the pictures are missing. And I don't know whether you folks have been photographed fairly late on the term, but a third of the pictures are missing. I already been putting some faces in my memory banks, but to match up names and faces so that I can hand out the problem sets individually, I need more photographs. So I'll do it one way or another next time, and we'll see some photographs, additional photographs, have appeared in the interim.

I've given you three problem sets so far, and they were really just for fun as well as to get you thinking about the right sorts of things, but there were little puzzle. I'm, unfortunately, going to have to tell you from now on no more cutesy little puzzles. You're going to get real problem sets, long, tedious, drudgery. They're optional though, so that's the saving grace.

So I'm going to pass it around the problem set 4. And this asks you to demonstrate some of the things that we talked about last time and convince yourself that they really work and ask you to make some simple applications of Miller-Bravais indices to two-dimensional lattices.

And then finally, the last problem is to get you thinking about patterns. And I've asked you to identify the lattice points and the symmetry elements in two patterns. I'm almost willing to wager that not one of you will get it completely right, that there's going to be some little thing that you missed or did wrong.

At one time, I bet a bag of potato chips for everybody in the class against the class wagering against me one bag of potato chips. Well, I'm going to watch my weight, and I've won that invariably, and I really can't eat that greasy sort of stuff.

In any case, you'll see that in the case of symmetry and patterns it is immensely simplified when you know exactly what to look for, when you know the number of possibilities was finite,. And when we're through with this, you can ask just one question about a pattern and then know exactly what the symmetry of that pattern has to be and exactly where to look for everything else.

I can guarantee you that by the time you finish this class you will never sit in a bathroom and stare at the tile floor and see it in exactly the same way again. I think that's a good observation because I like to think that that's what education is all about, not sitting in the bathroom, but seeing things differently after you've had the experience than you did before. That's what education is all about. So have fun with the patterns. And again, it will become-- one seat up here.

Today then we're about ready to embark on an adventure. As I said last time, we'll develop two-dimensional symmetries first because there are relatively few of them, and one could do this rigorously and in great detail. We're going to have to touch on things more lightly when we get to three-dimensional symmetries so we can finish at a decent point in the term.

Let me remind you that we have so far identified two types of symmetry that can exist in a lattice, onefold, twofold, threefold, or sixfold rotation axes. Any number of different rotational symmetries are possible, but if you're going to want them to be compatible with a lattice, you must restrict the rotation axis to one of these five, including a onefold axis, which is no symmetry it all.

And in two-dimensions, besides translation, we saw the operation of reflection. And now we're going to begin to put things together and make elaborate combinations. The first thing I will ask is can we have more than one of these symmetry elements present and operating about the same locus at the same time.

The answer to that would be, why not. Because if we look at a sixfold rotation axis, that's really what a twofold rotation axis does combined with what a threefold rotation axis does. And we plot these on top of one another, and what we end up with is the arrangement of motifs that is generated by sixfold axis. So in a sense, a sixfold axis is a twofold axis superimposed on a threefold axis.

That kind of a naive way about thinking of this though. What we're really saying is that what a sixfold axis has is the set of operations 1 A 2 pi/ 6, A twice 2 pi /6 -- that would be 120 degree rotation-- and A pi, A 5 pi/6. And there's 1, 2, 3, 4, 5. And we're missing one. 60, 120, 240. So that would be A twice 2 pi/3, 4 pi/6

So what we're really saying is a sixfold axis consists of these six elements, and these elements constitute a group because I can combine any two of these rotation operations and find something that's already a member of the set.

By saying that a sixfold axis consists of a threefold axis sitting on top of a twofold axis, what we're saying is that there's one collection of elements here, namely identity, or same thing as A 2 pi and A pi, that is a subset of these, or we could say a subgroup. So what we're doing in saying that these two sets of rotation operations exist simultaneously, we're saying that the twofold axis is a subgroup of the sixfold axis.

And similarly, we can separate out three other elements, a onefold axis, same as A 2 pi; a 120-degree rotation, A 2 pi/3; and a 240-degree rotation, A 4 pi/3. And these three operations, a group of rank 2, is what constitutes another subgroup, a subset of elements which by themselves satisfy all of the requirements of a group.

So this is one example of how we can have more than one symmetry element, a set of operations existing about the same locus and space. This is not how one would go about the deriving these higher symmetry however. Because what we have to do is to add something to the set, something that's called an extender, and then show that all the requirements of a set being a group is satisfied; namely, that a combination of any two elements in the group is also a member the group, that for every operation an inverse exists, and the identity operation is a member of the group. So you can build up more higher symmetries, more complex symmetries by adding some operation called an extender and then taking all of the products of these elements and see what new operations arise.

So we can have more than one symmetry element operating about the same locus and the set of individual operations that it embodies. And here sitting all by itself is a mirror plane. Why don't we combine a mirror plane with the rotation axis?

And we've already seen examples of that. For example, in the square pattern that these tiles make up, there's a fourfold axis in the middle of each of the squares that are also mirror planes that pass through that location.

So let me look at a combination that is a little simpler to handle. And what I'm going to say is here sits a mirror plane with a reflection operation that I'll call sigma 1 for the individual operation.

And let me say that I combine now in that space a second reflection operation about a line that intersects the first one. And the question I'm going to ask now is happens when I take a first motif, number 1, reflected in the locus of the first mirror plane to get number 2, and then reflect that a second time in the locus of the second mirror plane to get one that sits up here.

So the question is now what is the operation sigma 1 followed by the operation sigma 2 equals to? We can almost answer by the process of elimination. If this one is left handed, reflection changes the polarity, so number 2 is right hand. And if we reflect a second time, the right-handed one goes to a left-handed one.

So what we're asking is how do I get from one left-handed motif to another left-handed motif? Of the three operations that exist in a two-dimensional space could be translation. But, clearly, I can't take the first one and slide it parallel to itself and make it coincide with the third. So translation would not change chirality, but that won't work. What's left? Rotation. Yes. You want to clarify something first?

AUDIENCE: That third one, it looks kind of left-handed. [INAUDIBLE].

PROFESSOR: Oops. It does. It does indeed. Sorry about that. When you've got your nose poked right in these things, it's easy to overlook it. Yes. Absolutely right. That should be over canted over a bit more like I had it the first way. Very good thank you. I don't want to proceed much. Yes?

AUDIENCE: What did you [INAUDIBLE] axis [INAUDIBLE]?

PROFESSOR: We could do that. And let's do it, and we're going to encounter this sooner or later. So we have a first mirror plane reflect from a first one, which is right handed, to second one that's left handed. And then define a second locus, and that one would take the second one and move it over to here, the same distance on the other side of this mirror plane.

And I'll let you answer the question since you asked it. How is the first one related to the third one?

AUDIENCE: Translation.

PROFESSOR: Translation. Yes.

AUDIENCE: It's translated.

PROFESSOR: Exactly So if this distance is delta, the first and the third are related be a translation which is twice delta. So that would be in addition where we'll eventually want to consider when we have patterns that are based on a lattice. But right now, I want to consider symmetry operations that leave at least a point in space and move perhaps a line in the case of the mirror plane.

But back to the original question, what is this first reflection followed by a second reflection? We've just thrown out translation. That won't work unless the mirror planes are parallel to one of those. The only thing left is rotation. And in point of fact, even looking at this diagram, you can get from the first one to the third one by a rotation.

So the answer to the question is that a combination of two reflection operations is a rotation operation about the point of intersection of the mirror planes. But we can go further and say exactly how large this rotation is.

Let's draw an extra line in here. And let's say that this angle is alpha. And if I repeat this by reflection, that angle is also alpha. Let's label this angle beta. Repeat that by reflection, and that angle must also be beta. The angle between the two mirror planes is the angle mu. And clearly mu is equal to alpha plus beta.

So the answer to the question quite generally is have two mirror planes intersect at an angle mu, and their successive operation is going to be equivalent to the net result of rotating through twice of the angle between them mirror line. So we don't have to think about that anymore now. We know if we combine to mirror planes we get a net rotation.

So that is a basic result, and that's a first example of something that I'm going to call by my own pet term. I'm going to call them combination theorem, the theorem that tells you what you get when you combine two operations. And what this is going to give us is a way of filling in one box in the group multiplication table.

So knowing what two mirror planes combined at an angle should be, I can now combine the mirror planes at angles which gives me a rotation operation which is one of the ones that's allowed for a lattice. In general, if I'm not yet putting these symmetry elements in a lattice, I could combine them in such a way such that I got a 17-fold rotation as the angle. And then the angle between the mirror planes would be half of 2 pi/17. Could be a lovely symmetry, nice for a pendant or a ring, but not for a lattice and not for a crystal.

So I'm going to want to combine mirror planes at angles that correspond to rotations that are compatible with the lattice. So what I could do is take a first reflection locus, sigma 1. And if I want the rotation angle to be 180 degrees for a twofold axis, I would want the second mirror plane with the operation sigma 2 to be at one half of pi with respect to the first.

And for this specific combination then, reflection in the first mirror planes followed by reflection in the second mirror planes to give me a third one, is going to be equivalent to-- Oops. Back here-- it's going to be equivalent to a twofold axis. So sigma 1 followed by sigma 2 at an angle one half of pi is going to be equivalent to the rotation operation A pi.

This is not yet a complete pattern because here's a mirror planes that wants to reflect this motifs as well. So let's just reflect number 3 across the mirror planes 1. And then we're going to get a fourth object. Now let's show that this constitutes a group.

We've got the identity operation doing nothing. We've got the operation sigma 1. We've got the operation sigma 2, and we've got the operation a pi. I'll put the same operations again in the vertical column. Here's operation 1 doing nothing, sigma 1, sigma 2, and A pi.

So doing nothing and following it by one of these four operations just gives you the same thing back again. And doing these operations followed by 1 will give me the same operation back again.

If I reflect twice across sigma 1 from left to right and right to left, I'm back to where I started from. So sigma 1 followed by sigma 1 is the identity operation. Sigma 1 followed by sigma 2 is what we just did. That turns out to be the operation A pi. And sigma 1 reflection and following that by A pi is the way in which I get to the fourth one. So that is going to be equal to sigma 2.

Well, let me rips through the last ones fairly quickly. Sigma 2 followed by signal 1. Sigma 2 followed by sigma 1 is the same as A pi. Sigma 2 followed by sigma 2 gets us back to where we started from. That's the identity operation. Sigma 2 followed by A pi reflect from here to here and then rotate. That's the same as sigma 1.

And the final sequence, rotation followed by reflection would give me sigma 2. Rotation followed by sigma 2 will give me sigma 1. And doing the rotation operation twice is the same as the identity operation.

So are the group postulates satisfied? Yes. The combination of any two elements is a member of the group. For every operation, an inverse exists. And we can answer that question very easily by merely looking at the column under a particular element, and somewhere we find the identity operation. So sigma 1 is its own inverse. So, yes, an inverse exists for every operation. And then finally, identity is a member of the group.

So everything's lights up. Bells ring. This set of four elements is entitled to call itself a group. And in particular, it is a group of rank 4 because there are four elements in the group.

I show you a cool thing. I noticed that the number of objects in the pattern, the number of motifs in the pattern, is exactly the same as the order of the group. Why? Because these four operations tell you how to get from any operation to the remaining three and tell you how to get it into itself. So identity relates it to itself. Sigma 1 reflects it to here. A pi rotates it down to here. Sigma 2 reflects it down to here.

So there's always a one-to-one correspondence between the elements that are in the group and the rank of a group and the number of objects that are in the pattern. Yes, sir?

AUDIENCE: How do you show if the inverse exists again?

PROFESSOR: The inverse exists if I can find something that combined with an element in my basic set gives me the identity operation. So it's the operation followed by its inverse has to be the same as doing nothing.

So if I look under each element and find the identity operation, then sigma 2 is its own inverse. If I look under A pi, A pi is its own inverse. This is generally not the case. This is a very simple symmetry.

So really we can work with commas and little figures, and that's one way of getting these symmetries. But group theory I think you can see is a very nice, very elegant language for describing some characteristics of these patterns.

Now little bit of jargon. As if this material were not confusing enough, there are two different languages that are used to denote these unique combinations. This notation where we simply have a number for a rotation axis or an m for mirror plane is something that was first proposed by two mathematicians, Hermann and Mauguin. And this was adopted as the preferred notation in the international tables, that hefty tome that I brought it on the first day of classes. So this is also referred to synonymously as the international notation.

Let me put these things out horizontally. For rotation axes by themselves, the symbol in Hermann notation is simply n. And for lattice, n-- as we've seen many times-- is restricted to what in Hermann-Mauguin notation would be either 1, 2, 3, 4, and 6.

There's another type of notation that is use quite frequently by condensed matter physicist, and this is called the Schoenflies notation after one of the mathematicians who was the first to derive the three-dimensional symmetries.

This was a curious thing. There were three people in different parts of the world about the turn of the century-- and this was before email or even air mail-- who were trying to derive the three-dimensional symmetries. Schoenflies was one of them. And all three of them got to the same place at about the same time, and the results were obtained. But each one used his own method and had his different notation.

Schoenflies was one of the people who did this. Schoenflies was a mathematician and based his notation on group theory. Something called a cyclic group is what the C stands for. And a cyclic group is one in which all elements are power, so to speak, of some basic operation.

For example, a fourfold axis consists of the set of operations A pi/2; a 90 degree rotation; A pi, which can be written as A pi over 2 squared; A pi over 2 squared; [? then we need ?] A pi/2; and A pi/ 2 again; A 3 pi/2, which can be viewed as doing A pi/2 three times; and finally, the identity operation, which is the same as A 2 pi, which is equivalent to doing the basic operation A pi/2 four times.

So the group of rank 4, there are four elements in the group, each one is a power of the basic operation A pi/2. So a rotation axis by itself as a cyclic group. Schoenflies indicates these generically by C subscript n, where n is the rank of the axis. Hence, this is C1. This is C2. This is C3. This is C4. And this is C6.

A mirror plane by itself is indicated m in the international notation. A mirror plane is also a cyclic group. There are two operations, reflection and reflection back to where you came from. Doing the reflection operation twice is the same as doing nothing.

So Schoenflies also call this one C. And the one thing that is without meaning in English is his subscript S. And Schoenflies was German, and the S stands for spiegel, which is the German word for mirror. We have in many cities a paper that's called the *Daily Mirror.* In Germany, there's a paper called *Der Spiegel.* So they use it the same way in everyday life as well as in notation. So C sub S in the Schoenflies notation is a mirror plane. Cyclic group the S stand for spiegel.

For the combinations of which we have seen only one, in the Schoenflies notation these are all of the form Cnv, a rotation axis Cn with a vertical mirror plane passing through it. So this one, for example-- we've yet to look at the others-- would be C2v. So we'll have to draw the first, and then I'll give you the complete set. So this is a rotation axis with a mirror plane passing vertically through it.

Let's do a few more. And I think having done a couple in great detail, we'll see what the others will look like quite readily. Let's take a fourfold axis and add to the rotation operations A pi/2, a first mirror plane that I'll label sigma 1. I can permute the order of operations and say that sigma 1 followed by the rotation operation A pi/2 is going to be equal to a second reflection operation that is equal to one half of pi/2 away from the first.

So I'm claiming that if I reflect in the first mirror plane and then rotate by 90 degrees that should be another mirror plane at 45 degrees to the first. So let me do exactly what I advertised.

Here's the first, one right handed. Reflect across to get a second one, which is left handed. Reflect, then rotate, and that would bring my left-handed one up to this location here. So here's 3, and it's left handed. And lo and behold, just as advertised, I get from the first one to the third one by reflecting across a mirror plane, which is one half of pi/2.

If let these symmetry elements operate on each other, what I'll end up with is a set of mirror planes at 45 degree intervals. And what I'll have is a pair of objects hung in the same fashion at every mirror plane.

Is that too fast? You want to go through that a little slower? Oh, go ahead say, go through it more slowly. Everybody's afraid to say, yeah, yeah, and been seem like a class dummy. Do you want me to do it more slowly? I see people still writing, so I think what you'd rather have is me be quiet for a bit while you catch up to where I am.

In the Schoenflies notation, this lovely thing here is C4. It's a rotation axis of rank 4 with a vertical mirror plane added to it. And the international notation, now the Hermann-Mauguin notation, is just a running list of the individual symmetry elements that are present.

And now I really are going to have a mouthful. This is a fourfold axis, so the symbol for that is 4. This is a mirror plane. This is a mirror plane. This is a mirror plane. This is a mirror plane. This is a mirror plane. This is a mirror plane. That's a mirror plane, and that's a mirror plane. So it looks as though I should call this 4mmmmmmmm, which comes 4mmmmmmmm. Well, that is a typical reaction. It's a lovely symmetry. But that is a mouthful.

So it isn't really necessary to give all these m's So the international notation is a running list of the independent symmetry elements, and that's the new wrinkle that I'm introducing. It's a running list of the independent symmetry elements. And all these mirror planes are just different sigma that exist as operations in the group.

But how many different kinds of mirror planes are they? Well, there are two different kinds of mirror planes, both in terms of the way in which they function in the pattern. The two motifs related by reflection hang close to this mirror plane, but they're widely separated for this mirror plane. So the motifs do different things relative to those two kinds of mirror planes.

Another way of asking what's independent is if I start with this 1 mirror plane and repeat it by 90-degree rotations, I'll get these 4 mirror planes 90 degrees away. So they are not independent in that these mirror planes are all related by the rotational symmetry that's present. You don't get this mirror plane in any fashion other than saying, if I combine the rotation operation with this reflection operation sigma, the net result is this reflection plane.

So there are two mirror planes that are distinct in this arrangement of symmetry elements, distinct in the sense that they function in different ways in the pattern; distinct in the sense that no other operation that is present will throw these two operations into one another.

Another example, and this is in fact symmetry 4mm, is the square tile. If you look at the mirror planes there, they are 45 degrees apart. But one of those mirror planes comes out normal to the edge of the tile. The other mirror plane comes out of the vertex of the tile. So they are different in the way they function in the space which has this symmetry.

So, mercifully, we don't have to call this 4mmmmmmmm. We drop the last 6 m's, and this one is called simply for 4mm, 2 kinds of mirror planes with a fourfold axis. If you're familiar with the way in which these were derived, the symbol tells you exactly what you've got. So it's a very useful notation for these symmetries.

Let me pause here, give you a chance to catch up. Yes, sir?

AUDIENCE: So wait, all you did here is take two separate mirror planes at an angle of 45 degrees between, and you ended up with a fourfold symmetry.

PROFESSOR: Exactly.

AUDIENCE: But you didn't put on the fourfold symmetry, it was just what fell out of it?

PROFESSOR: No. I Got this mirror plane to begin with by combining this reflection operation sigma with the operation A pi/2, and that's where this operation came from.

But a general theorem that says if I have a rotation operation A alpha and combine it with a reflection operation sigma, the combined effect is a reflection operation sigma 2, which is alpha/2 away from the first.

So, again, showing you for this now rather messy diagram, if I start with the reflection operation that takes 1 of them throws of the 2 and then rotates 2 up to location 3, the way I get from 1 to 3 in one shot is to reflect in this locus sigma 2. Yes, sir?

AUDIENCE: You could have started with any one of those two and got the third.

PROFESSOR: Absolutely. Absolutely. The operations that are present operate everything in the space. So when I say that there's a mirror plane here that relates this one to this one, it also relates this one to this one, this one to this one, this one to this one, that mirror planes operates on everything. You can't say that a symmetry operation grab this little packet of space and moves it to another packet removed from it. To say it's a symmetry of a pattern or of a crystal, it has to leave everything invariant. OK.

But, again, in terms of the language of groups theory, if you combine this pair of operations, then when you combine everything that you get pairwise, you will get in this case a total of, how many operations? How many operations are in this pattern? I said a moment ago the rank of the group is the number of operations that are present. It's the number of objects that are in the group because each operation in a group tells you how to get from anyone to all of the other. So, 2, 4, 6, 8, this is a group of rank 8, and there are 8 operation that are present.

And I can rattle them off quickly. Four operations, identity, 90, 180, 270 for the fourfold axis. That's 4. This mirror plane. This mirror plane. The mirror plane. And this mirror plane. That's 8. Four for rotation. Four for reflection. Other questions?

All right. Let me then wrap up this quickly and get onto something new. Did I hear the hiccup or a question? No. It was a hiccup. Not a yawn I hope.

Let me do the highest symmetry of all in two dimensions, and this is if I take a sixfold axis and combine it with a mirror plane. So here's the first operation sigma. This one is something of a bear to draw. This is sigma 1. Sigma 1 followed by the operation A 2 pi/6 should be equal to a reflection that is an angle one half of 2 pi/6 30 degrees away from the first. So this will be sigma 2.

So if I reflect from 1 to 2 and then rotate by 60 degrees, I'm going to get one that sets up here. And the way I get from number 1 to number 3 in one shot is by a reflection sigma 2.

Now if I draw in all of those are mirror planes when they are repeated by the sixfold axis, I shouldn't have given this my 6mmmmmm treatment because there are 12 mirror planes. Two objects hanging on this one in one fashion spaced in a different way that are belly to belly. They're back to back here, so it does something different in the pattern. A pair hanging here. A pair hanging in here. A pair hanging and here. A pair hanging in here. And finally, a pair hanging in here.

So there are a total 2, 4, 6, 8, 10, 12 motifs in this pattern. Pairs hanging on mirror planes that are 30 degrees apart and hanging disposed and pointing in different ways on the adjacent mirror planes. So this is one that we would call in international notation 6mmm and in Schoenflies notation C6v.

So we're making extraordinary progress here. The one that I did for you initially was C2v, and that's 2mn in international notation. The only one that I left out to this point is a threefold axis compared with a vertical mirror planes.

And let's start with the operation A 2 pi/3, combined with that a first reflection operation sigma 1 that passes through it. And for reference, I'll drawn in some lines that are separated by intervals of 60 degrees.

So this reflection from 1 to 2 followed by a rotation 120 degrees should give me this one as number 3. The first is right handed. The second is left handed, and the third stays left handed.

And lo and behold, the way I get from 1 to 3 directly is by a reflection sigma 2 across a mirror line that is-- I'm sorry. This is number 1 down here-- across a mirror line that is 30 degrees away from the first.

If I would complete the pattern, reflect this one across to here, take this pair and reflect it or rotate it up here, and I have six objects. So this is a group of rank 6. And it has characteristics that are quite analogous to those we did for the other rotation axes in all respect except one.

The international symbol for this combination is C3v. We've got a threefold axis. We've got the mirror plane that we added. And then we've got a mirror plane that is 30 degrees away from the first-- I'm sorry-- 60 degrees away from the first, one half of 120 degrees.

I claim that this is not the proper symbol because the mirror planes that are listed should be the symmetry-independent, distinct sort of mirror planes. And is that the case with this one? The answer is no. It isn't.

Here is one mirror plane. It's got a pair of motifs hanging on it. This mirror plane here is a mirror plane that has a pair of motifs hanging it. Same is true of this mirror plane. So all six of these mirror planes are doing the same thing in the pattern. Each one has a pair of a motifs on either side of it in the same fashion. And I can get one mirror plane and the motifs hanging on it by a rotation of 120 degrees.

So there's only one kind of mirror planes, so we don't need that m. So all of the other rotational symmetries, m, a onefold axis with a mirror plane; 2mm, 4mm, 6mm have two kinds of mirror planes. C3v has only one kind of mirror planes.

And another way of showing that is if I look at a trigonal prism. It's got a mirror plane coming out of a corner and out of the opposite face. Mirror plane coming out of the corner and out the opposite face. Mirror plane coming out of a corner and out the opposite face. And that's exactly the rearrangement of symmetry elements here.

So each mirror planes when drawn in relative to a trigonal prism, which is a body that has this symmetry, each mirror planes does exactly the same thing, as opposed to the square tile or square prism that has one kind of mirror plane that comes out of faces and one kind of mirror plane that comes out of corners. So there are two different types of mirror planes.

Let's add them all up, summarize, and we've got the cast of characters that we can be use in deriving two-dimensional symmetries. We've got C1, which is a onefold axis; C2, twofold axis; C3, that's a threefold axis; C4, and that's a fourfold axis; C6, that's a sixfold axis.

Then we've got the additions that involve adding a mirror plane to a rotational axis. So this is simply m and CS in Schoenflies notation. Then we added a mirror plane to a twofold axis. We've got C2 with a vertical mirror plane, 2mm; C3v; and not 3mm, but only 3m; C4v, and that's 4mm; and C6v, and that's 6mm. Add them all up, and there are 10 unique possibilities, no more, no less. Yes, sir?

AUDIENCE: I'm missing a point. Why is there the mirror plane sigma 2 rather than rotating sigma 1 is not different? How do you?

PROFESSOR: The same thing is going on, on this mirror plane as on this mirror planes, on this mirror plane. Or if you like to see it in terms of a geometric solid, the mirror planes here in a trigonal prism all of them do the same thing. They come out of one corner and out of the opposite edge. Each of these has one pair of objects hanging on it. And that is true of all three of them.

And if you like, this end of the mirror plane here is nothing more than one that's related to the first one by one of the rotations of the threefold axis extended back in the opposite direction. So the mirror plane doesn't just work up here. It works down here as well. It works all through the space.

So any way you want to look at it and whatever terms work for you, each of these mirror planes is the same, but they have a different type of end to them. There's something different hanging at one end than at the other end. Think of it in terms of actual physical object.

AUDIENCE: [? What about the ?] case of the fourfold [? there? ?]

PROFESSOR: In the fourfold, one comes out of the face. One comes out at the edge. The motifs are oriented and spaced at a different distance from each of those neighboring mirror planes.

So these two are different. The fourfold axis never rotates this one into this one. These two are not different because the threefold axis rotates, if you do it twice, this end into this end of what is one in the same mirror plane as what's down on here.

So what's confusing you I think is that there's a polarity to the mirror planes. Both ends of the mirror planes do not have the same disposition of motifs on them, but there's nothing that says that this has to be the case. The motifs could just be at one end.

And that, if you think about it a little bit if it help, figure out-- not on company time though, but on your own time-- what a fivefold axis does. And a fivefold axis is a non crystallographic symmetry. But it turns out that C5v, which is a 5 with an m, has only one kind of mirror plane in it as well and for exactly the same reason. A regular figure that has the symmetry is a pentagon. A mirror plane comes out a corner and out of the opposite face. And that's true for all of the mirror planes that are in there and separated by one half of 2 pi/5.

AUDIENCE: That's just whether or not the [INAUDIBLE] axis is [INAUDIBLE]?

PROFESSOR: Yeah. That's what I said. Yeah. So before I go through all of the rotation axes with vertical mirror planes, the international notation would be 2mm, 3m, 4mm, 5m, and 6mm, 7m. For the odd symmetries, there's only one kind of mirror plane in a pattern, in the tile, or whatever you want to have.

So there are 10 distinct possibilities, and these are called the Point Groups. Why? Because they are clusters of symmetry elements about at least one point that's fixed and embedded immovably in space. They're called groups because, as we've seen for one simple example, the collection of operations follows the postulates for the set of elements which we have defined as a group.

More specifically, we could call these the 10 two-dimensional crystallographic graphic Point Groups, which is more of a mouthful. But it emphasizes the fact that there are lots of two-dimensional Point Groups, but the two-dimensional crystallographic Point Groups are those that involve rotational symmetry that are compatible with a lattice. Hence, they are crystallographic. But the number of Point Groups is actually infinite if you include the ones that are not compatible with translation.

All right. I think that's probably a good place to quit. And guess what? My internal clock has told me that this is the time for our break. This is one crossroads in our development. And what we'll do next for the faint hearted who may not want to have more of this stuff for one day than what we've just done, we're now going to do the final penultimate combination, and say we have also shown that there are 5 two-dimensional lattices.

And the final step will be to say if we have a pattern that has symmetry and is based on translation, we can obtain these by taking each of the 10 crystallographic Point Groups in turn and dropping them into each of the five lattices that can accommodate them.

We would not try, for example, to take a Point Group like 4mm and try to drop it into the hexagonal lattice. It's not going to fit. But there will be two, maybe three ways in which we can add a given Point Group to one of these lattice types.

And what we've done when we finished is we have exhaustively derived the symmetries' translational lattices and symmetry operations of reflection and rotation that are possible for two-dimensional crystal.

You might say, well two-dimensional crystal, I've heard that people who do thin film work to make monolayers and really make a two-dimensional crystal. That's not something you could say only a few years ago. But why do we worry about two-dimensional symmetries if we're not to be wallpaper designers? Well, actually, I'll give you one example.

One of the difficulties you had in conveying the nature of a crystal structure is taking something that's fairly complicated in three dimensions and getting it onto a two-dimensional sheet of paper. And what you invariably do is you project the contents of the unit cell down along one of the cell edges. And what you have then is a two-dimensional crystal structure with a lattice and symmetry.

So you'll see two-dimensional representations of translationally periodic arrangements of atoms quite frequently when you look at projections of actual crystal structures, which is the only reasonable way to convey the information of any structure once you get on beyond the baby stuff of sodium chlorides, zinc, sulfide and body-centered iron.

OK. I'm going to pause, suck in air. I am happy to turn you loose for 10 minutes and will meet here again, let's say, at 10 after 3:00.