3.60 Symmetry, Structure and Tensor Properties of Materials

Problem Set 7

(1) For values of n from 1 to 8

(a) Sketch representative patterns of motifs for $\bar{n}$ and $\tilde{n}$. It is suggested that you use open and filled circles to designate motifs that are "down" and "up", respectively, to avoid the introduction of false symmetry in the patterns.

(b) Decompose each of the patterns into combinations of simpler one-step operations where possible. For example, $\bar{3} = 3 + \bar{1}$.

(c) Which rotoinversion or rotoreflection "axes" cannot be decomposed.

(2) Successive rotations A_α and B_β about intersecting rotation axes are equivalent to a net rotation C_γ about a third axis. The angles, β, and γ must be crystallographically-permissible rotation angles if the three rotations are to be operations in a crystallographic point group. Determine whether the following combination of operations is permitted.

	α	β	γ
(a)	$2\pi/6$	$2\pi/3$	$2\pi/2$
(b)	$2\pi/4$	$2\pi/3$	$2\pi/3$
(c)	$2\pi/4$	$2\pi/3$	$2\pi/2$
(d)	$2\pi/4$	$2\pi/2$	$2\pi/2$

Note that in order to fully describe a combination of rotation axes it is necessary to specify values for all <u>three</u> interaxial angles.

(3) For those of the above combination of rotation axes that are possible show, by means of a sketch, that $B_\beta \bullet A_\alpha$ is indeed equivalent to a net operation C_γ. (A convenient device is to use the corners of a polyhedron that conforms to the symmetry of the combination as the locations of the objects involved in the successive mappings.)

(4) It turns out that $\alpha = \beta = \gamma = 2\pi/4$ is <u>not</u> a permitted combination of rotation operations in Euler's construction. (Try calculating the angles if you are skeptical.) Yet, there are three orthogonal 4-fold rotation axes present in point group 432. How come? Does this mean the operations in 432 cannot constitute a group? Please explain.

(5) Derive the point groups (non-crystallographic) that may be obtained by adding as extenders

(a) a horizontal mirror plane
(b) a vertical mirror plane
(c) a diagonal mirror plane
(d) an inversion center

to the combination of rotation axes 522.

For each addition, please

(i) indicate the steps and theorems used in your derivation (eg. $\sigma_1 \bullet A_\pi = \bar{1}$)

(ii) sketch the resulting combination of symmetry elements

(iii) state the international and Schönflies symbol for each resulting point group.