3.60 Symmetry, Structure and Tensor Properties of Materials

Problem Set 15

1. Derive the form of the piezoelectric modulus matrix d_{ij} which is required by an orthorhombic crystal with symmetry 222.

2. The "longitudinal piezoelectric effect" measures the charge per unit area (equivalent to the normal component of $\vec{P}$) developed on the surface of a plate when it is subjected to a uniaxial tensile stress. That is, we cut a plate normal to some direction x_1', apply a tensile stress σ_1' and measure P_1'. The magnitude of the longitudinal piezoelectric effect is therefore given by

$$P_1' = d_{11}' \sigma_1'.$$

Using the appropriate form of the piezoelectric modulus matrix (supplied on a handout) derive an expression for the variation of the longitudinal piezoelectric modulus as a function of direction for a crystal with

(a) symmetry $\bar{4}3m$ (e.g., the sphalerite structure type)

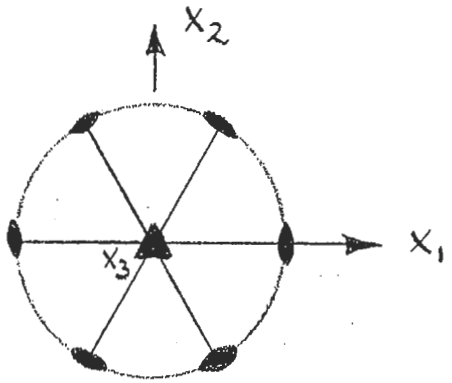
(b) symmetry 422

For each of the results which you obtain, sketch the variation of d with direction and show that the surface conforms to the point group of the crystal.

3. The piezoelectric moduli for quartz, SiO_2, are

$$d_{ij} = \begin{pmatrix} -2.3 & 2.3 & 0 & -0.67 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.67 & 4.6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} 10^{-12} \text{ coulombs/newton (mks units)}$$

where $P_i = d_{ij}\sigma_j$. Quartz is hexagonal, with symmetry 32. The tensor above has been defined relative to the following coordinate system.



(a) What is the magnitude and direction of the polarization developed when the crystal is subjected to

(i) a tensile stress along x_1

(ii) a tensile stress along the 2-fold axis in between x_1 and x_2

(iii) a tensile stress along x_2

(iv) a shear stress σ_{23} about x_1

(b) What, in general, is the result of any pure tensile stress applied to the crystal? In quartz the 2-fold axis is referred to as the "electric axis". Can you guess why?

(c) What is the strain which results from the application of an electric field of 10^5 volts/meter along x_1? (This is a hefty field!)

(d) Would you consider the magnitudes of the polarizations or strains produced in the piezoelectric effect to be large?