3.185 Problem Set 4

Heat Conduction

Due Monday October 6, 2003

1. Thermal properties and optimal materials selection (27)

   In many situations, product designs include parts whose only function is to conduct or resist the conduction of heat. Materials for these parts are thus chosen entirely on the basis of their thermal properties. Select the best material from the list provided for each of the following applications.

   (a) Heat shield sandwiched between a hot body and a cold one which minimizes the steady flux between them. (4)

   (b) Heat shield which protects something from short, intense bursts of heat (long timescale is needed). (4)

   (c) Cheap (i.e. not diamond) temperature sensor, in which short timescale of heat conduction is necessary for rapid response. (4)

   (d) Light heat reservoir which must hold as much heat as possible per degree C per unit weight. (4)

   (e) Heat sink for a semiconductor device, which must minimize temperature difference for a given flux. (4)

   (f) Heat sink for melt spinning, in which liquid metal is injected against a rotating heat sink where it is solidified as rapidly as possible, so the material must conduct heat away from the surface quickly. (Hint: evaluate the flux through $x = 0$ in an erfc-like unsteady conduction problem. Diamond is not an ecomonically viable option.) (7)

   Candidate materials:

Material	k, $\frac{W}{m \cdot K}$	ρ, $\frac{g}{cm^3}$	c_p, $\frac{J}{kg \cdot K}$
aluminum	238	2.7	917
copper	397	8.96	386
gold	315.5	19.3	130
silver	425	10.5	234
diamond	2320	3.5	519
graphite	63	2.25	711
lime (CaO)	15.5	3.32	749
silica (SiO_2)	1.5	2.32	687
alumina (Al_2O_3)	39	3.96	804

2. Layered furnace wall and British units (22)

   A cylindrical furnace wall consists of an inner graphite layer 18 inches (1.5 feet) thick, followed by an outer layer of insulating brick four feet thick, both surrounding a cylindrical container 15 feet high (on the inside) with an inner diameter of 20 feet. The heat transfer coefficient between the outer wall and the environment is 4 BTU/hr·ft^2·°F, that between the molten metal and the inner wall can be assumed infinite (it's a lot bigger), the thermal conductivity of the brick is 16 BTU/hr·ft·°F, and use the value for graphite conductivity in the table in problem 1 times $0.557 \frac{\text{m}\cdot\text{K}}{\text{W}} \frac{\text{BTU}}{\text{hr}\cdot\text{ft}\cdot{}^\circ\text{F}}$.

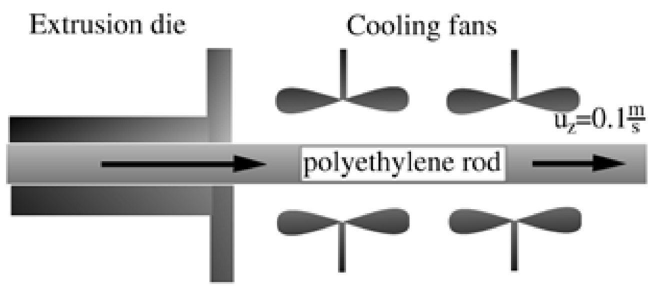
   (a) Calculate the total heat loss in BTU/hr from molten metal at 2000°F to an environment at 70°F. (You may neglect the corners of the cylinder, but include the top and bottom.) (8)

   (b) How would your solution to part 2a would change if you include heat conduction in the corners? (*I.e.* is your answer an overestimate or underestimate?) (5)

   (c) Calculate the temperature at each of the interfaces in the radial direction. (9)

   1 BTU (British Thermal Unit) of heat raises the temperature of 1 pound of water by 1°F.

3. Polymer extrusion and thermal stress (27)



   A cylindrical high-density polyethylene rod 0.02 m in diameter is exiting an extruder at a uniform rate of $0.1 \frac{\text{m}}{\text{sec}}$ and a temperature of 160° C. It is cooled by fans to room temperature which is 40° C, with a heat transfer coefficient of $130 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$.

   Thermal stress is roughly proportional to temperature difference between the surface and center of the rod, so we want to estimate that temperature difference.

   You may neglect thermal gradients in the lengthwise direction, and assume steady-state, so the energy transport equation will be

$$r u_z \frac{\partial T}{\partial z} = \alpha \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)$$

   HDPE data: $k = 0.64 \frac{\text{W}}{\text{m}\cdot\text{K}}$, $\rho = 920 \frac{\text{kg}}{\text{m}^3}$, $c_p = 2300 \frac{\text{J}}{\text{kg}\cdot\text{K}}$.

   (a) Calculate the Biot number for this situation using the radius as the lengthscale. (5)

   (b) Calculate the Fourier number at $z = 0.33$m, $z = 1$m and $z = 3.3$m (hint: convert z to time). (5)

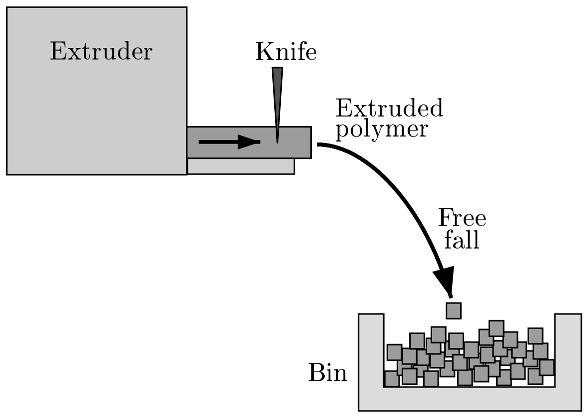
   (c) Use the graphs in Appendix F of W^{3}C to estimate the temperatures at the center of the rod and the surface at those three distances. Of these three, which gives the largest temperature difference between the surface and the center? (12)

   (d) If your 3.11 thermal stress calculation tells you the temperature is too non-uniform and will lead to product defects, how can you correct this? That is, what simple design modification would make the temperature more uniform? (Hint: consider the shape of the T vs. r curves for various dimensionless numbers.) (5)

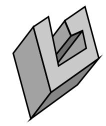
4. Cooling of a little plastic widget (24)

Little plastic widgets are made by extruding a polymer through a shaped die and slicing it off with a knife every time it reaches a certain length. These widgets then fall through the air and cool off as they do so, hopefully reaching an acceptable temperature when they land in the bin.

The process



The widget



Each widget has a maximum dimension of 5 mm (0.005 m), surface area of 2 cm^2 ($2 \times 10^{-4}\text{m}^2$), and volume of 0.05 cm^3 ($5 \times 10^{-8}\text{m}^3$).

Thermal data:

- Widget initial temperature: 160° C
- Ambient air temperature: 20° C
- Polymer thermal conductivity: $k = 2.0 \frac{\text{W}}{\text{m} \cdot \text{K}}$
- Polymer density: $\rho = 900 \frac{\text{kg}}{\text{m}^3}$
- Polymer heat capacity: $c_p = 2500 \frac{\text{J}}{\text{kg} \cdot \text{K}}$
- Heat transfer coefficient: $h = 40 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$

(a) Calculate the heat transfer Biot number of the widget, using the maximum dimension as the lengthscale. What assumption can you make about the cooling behavior? (5)

(b) The widget takes 10 seconds to fall (it's dropped from quite a height, and reaches its terminal velocity quickly). Calculate the Fourier number at the end of this fall, again using the maximum dimension as the lengthscale. (5)

(c) After that much time (10 seconds), estimate the temperature in the "center" of the widget (its maximum temperature). (9)

(d) How will your answer to part 4c change if a different polymer is used with half the thermal conductivity, but approximately the same density and heat capacity? (5)