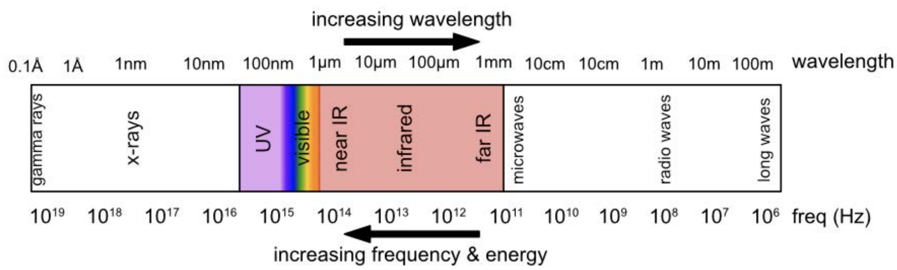
1 Waves and photons

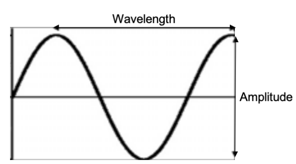
Photons are quanta of light: the energy of a photon with frequency ν (or wavelength λ) is given by the Planck-Einstein relation.

$$E = h\nu = \frac{hc}{\lambda}$$

Here, h is Planck's constant, and c is the speed of light. The electromagnetic spectrum is shown here:



Recall that the equality above came from the fact that frequency and wavelength are related through the speed of light as $c = \lambda\nu$. It's also important to remember that the energy of a photon is independent of the amplitude of the wave:



The energy is set by the wavelength (or equivalently, frequency); the amplitude of the wave is related to its intensity: this can be thought of as the number of photons of a certain energy.

Example: A radio station broadcasts at 100.7 MHz with a power output of 50 kW. How many photons are emitted each second? Recall that $1W = 1J/s$.

The energy per second that leaves the radio station is

$$50 \text{ kW} \times \frac{1000 \text{ W}}{1 \text{ kW}} \times \frac{1 \text{ J/s}}{1\text{W}} = 5 \times 10^4 \text{ J/s}$$

We can calculate the energy per photon with the Planck-Einstein relation:

$$E = h\nu = (6.626 \times 10^{-34} \text{ J*s}) \times (100.7 \times 10^6 \ s^{-1}) = 6.672 \times 10^{-26} \text{ J/photon}$$

Finally, the number of photons per second is

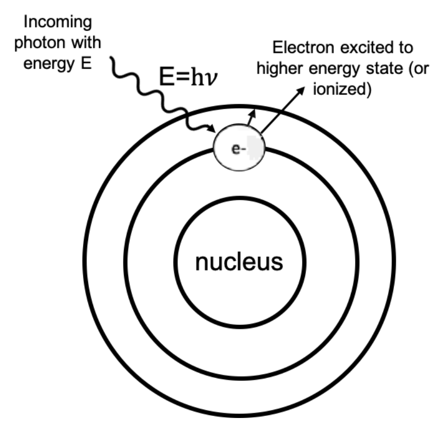
$$N = (5 \times 10^{-4} \text{ J/s})\frac{1 \text{ photon}}{6.672 \times 10^{-26} \text{ J}} = 7 \times 10^{29} \text{ photons/s}$$

2 Bohr model

The *photoelectric effect* tells us that shining light with enough energy on an atom can cause the emission of electrons. In 3.091, we'll use the Bohr model to model this process. In the Bohr model, an electron in a hydrogen atom can only live in discrete energy levels, which we label with n. The lowest energy level is the ground state, with $n = 1$. The change in energy corresponding to the transition from an initial energy level, n_i to a final energy level, n_f is given by

$$\Delta E = -13.6\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)[eV]$$

If a photon with just the right energy impinges on the atom, the photon can be absorbed and the electron excited to a higher energy level.



Conversely, if an electron relaxes from a higher energy level to a lower one within the atom, a photon with frequency $\nu = \Delta E/h$ is released.

Example: A power source emits 8×10^{18} photons/s at 10 W. Determine how much energy each photon has. Then, calculate what state an electron in the ground state of a hydrogen atom would end up in if excited by a photon with this energy.

To calculate the energy of each photon, we can use dimensional analysis to get to Joules:

$$E = \frac{10 \text{ J/s}}{5 \times 10^{18} \text{ photons/s}} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 12.5 \text{ eV}$$

If the electron starts in the ground state, we need to solve

$$\Delta E = 12.5 \text{ eV} = -13.6 \text{ eV}\left(\frac{1}{n_f^2} - \frac{1}{1^2}\right) \qquad n_f = 3.51$$

Since the electron must live in an integer state, it must either go to the third or fourth level. It would take more energy than we have to get to the fourth energy level, the highest accessible energy level is $n = 3$.