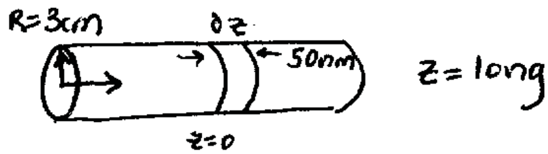
3.044 MATERIALS PROCESSING

LECTURE 7

Ex. 4: Friction Welding → geometry not always obvious without some calculations

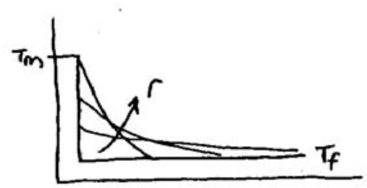
Problem Statement: Locally need to melt and join, not much heat away from the joint or "heat affected zone" (HAZ)

Geometry:



Boundary Conditions:

$$
\begin{aligned}
@z = 0, \text{symmetry:} \quad & \frac{\partial T}{\partial z} = 0 \\
@r = 0, \text{symmetry:} \quad & \frac{\partial T}{\partial r} = 0 \\
@t = 0, \text{in } \partial z: \quad & T = T_m \\
@r = R = 3\text{cm}: \quad & q_{\text{conv}} = h(T - T_f) \\
& \text{where } T_f = 25°\text{C}, \text{and } h = 10 - 20\frac{W}{m^2 K} \\
@z = \infty: \quad & T = T_f
\end{aligned}
$$



Date: February 29th, 2012.

Governing Equation:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

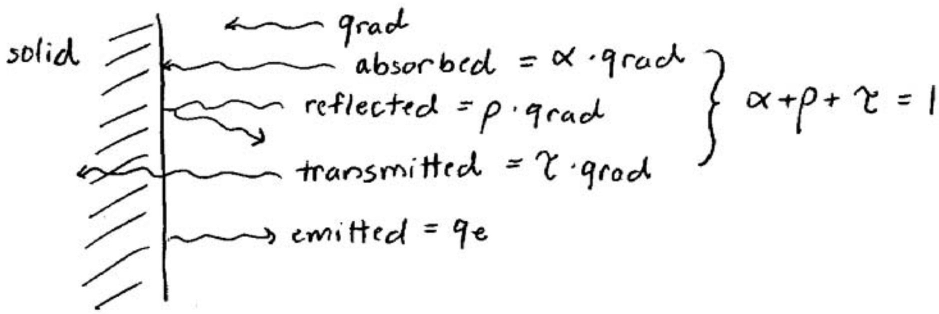
$$\text{Bi}_r = \frac{hL}{k}\text{, where h = 10-20, L =}\frac{R}{2}\text{, and k = 35}$$

$$= \textbf{small} \Rightarrow \text{no gradient in the solid } \underline{\text{ALONG r}}$$

Solution:

$$\frac{T - T_f}{T_m - T_f} = \frac{1}{2}\left(erf\frac{\partial - z}{2\sqrt{\alpha t}}\right) + \left(erf\frac{\partial + z}{2\sqrt{\alpha t}}\right)$$

Radiative Heat Transfer



In equilibrium: energy absorbed = energy emitted ⇒ $\boxed{\alpha q_{\text{rad}} = q_e}$

A **Black Body** is an idealized solid that

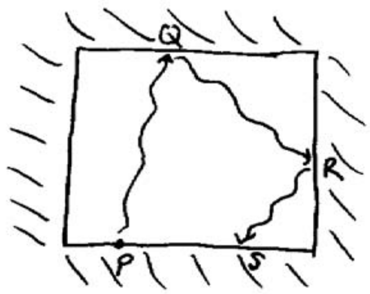
1. absorbs everything: $\boldsymbol{\alpha_b = 1}$
2. emits light perfectly (Planck, Stefan-Boltzman): $\mathbf{q_{e,b} = \sigma T^4}$
   ⇒ σ is the Stefan-Boltzman constant = 5.669 x $10^-8 \frac{W}{m^2K^4}$

Blackbody in equilibrium: $\mathbf{q_{rad,b} = q_{e,b}}$

Most solids and liquids are not "black" but "gray": $\mathbf{q_e} = \varepsilon \mathbf{q_{e,b}}$
⇒ ε = emissivity: a unitless fraction of blackbody emmitted flux

Another important note: $\alpha = \varepsilon$, for gray bodies emissivity = absorptivity

Is there a real blackbody?: Sort of... there are "cavities"



Light is emitted but cannot be lost:

At "p" emission

$$q_e = \varepsilon_{\text{wall}}\, q_{e,b}$$

At "Q" reflection

$$q_Q = \rho_{\text{wall}}\, \varepsilon_{\text{wall}}\, q_{e,b} = (1 - \alpha_{\text{wall}})\, \varepsilon_{\text{wall}}\, q_{e,b}$$

At "R" reflection #2

$$q_R = \rho_{wall}\, (\rho_{wall}\, \varepsilon_{wall} q_{e,b})$$

At "S" reflection #3

$$q_S = \rho_{wall}\, \rho_{wall}\, \rho_{wall}\, \varepsilon_{wall}\, q_{e,b}$$

After n reflections

$$q_n = \rho_{wall}^n\, \varepsilon_{wall}\, q_{e,b}$$

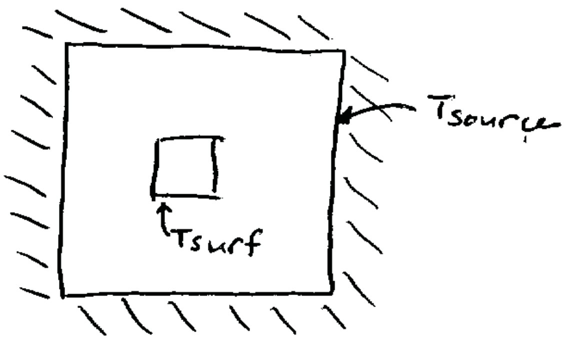
Total

$$\begin{aligned} q_e &= (1 + \rho_{wall} + \rho_{wall}^2 + \ldots)\, \varepsilon_{wall}\, q_{e,b} \\ &= \frac{1}{1 - \rho_{wall}} \varepsilon_{wall}\, q_{e,b} \\ &= \frac{\varepsilon_{wall}}{1 - \rho_{wall}}\, q_{e,b} \\ &= \frac{\varepsilon_{wall}}{\alpha_{wall}}\, q_{e,b} \\ &= q_{e,b} \end{aligned}$$

In a cavity:

$$\boxed{q_e = q_{e,b} = \sigma T^4}$$

What kind of boundary condition can we write?



Net flux at the object:

$$\begin{aligned} q_{net} &= q_{\text{emitted}} - q_{rad}\alpha_{\text{incoming}} \\ &= \varepsilon q_{e,b} - \varepsilon q_{\text{rad}} \\ &= \varepsilon\left(\sigma T_{\text{surf}}^4 - q_{\text{rad}}\right) \\ &= \varepsilon\left(\sigma T_{\text{surf}}^4 - \sigma T_{\text{source}}^4\right) \\ &= \varepsilon\sigma\left(T_{\text{surf}}^4 - T_{\text{source}}^4\right) \end{aligned}$$

Summary:

$$\begin{aligned} q_{\text{cond}} &= -k\frac{\partial T}{\partial x} \\ q_{\text{conv}} &= h(T - T_f) \\ q_{\text{net}} &= \varepsilon\sigma\left(T_{\text{surf}}^4 - T_{\text{source}}^4\right) \\ T^4 &: \quad \text{rapid onset, important at high T and irrelevant at low T} \\ T_{\text{obj}}^4 - T_{\text{furnace}}^4 &: \quad \text{if the object is much colder than the furnace then you can assume } T_{\text{obj}}^4 \approx 0 \end{aligned}$$

To compare convection and conduction use: **The Bi number**

To compare conduction and radiation use:

$$k\frac{\partial T}{\partial x} = \varepsilon\sigma\left(T_{\text{surf}}^4 - T_{\text{source}}^4\right)$$

$$\Rightarrow \boxed{\mathbf{M} = \frac{\mathbf{L}}{\mathbf{k}}\varepsilon\sigma\mathbf{T}_{\text{surf}}^{\mathbf{3}}}$$

If M > 10, then radiation is fast, conduction is rate limiting
If M < 0.1, then conduction is fast (no gradients)