3.044 MATERIALS PROCESSING

LECTURE 1

What is Materials Processing?

- A way to make materials **useful**: desired chemistry, shape, microstructure
- A way to give materials the desired properties

What Processes are included in copper production?

grinding → colloid / suspension → refining / reducing → casting → electrolysis → melting → casting → rolling (hot) → drawing

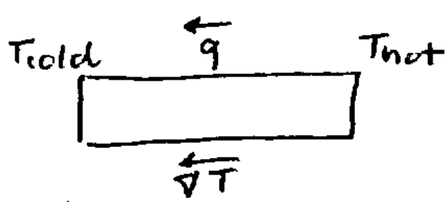
What thermodynamic variables do we have to work with?

T	P (or σ)	C (composition)
heat	beat (move matter)	mix
heat transfer	solid mechanics	chemical reaction
	fluid mechanics	phase transformation
		diffusion

Topics Covered in this Class

Part I: Heat transfer
Part II: Fluid flow
Part III: Combine all 3

Heat Conduction: Heat flows down the temperature gradient



Date: February 8th, 2012.

Fourier's Law: $\bar{q} = -k\overline{\nabla T}$,

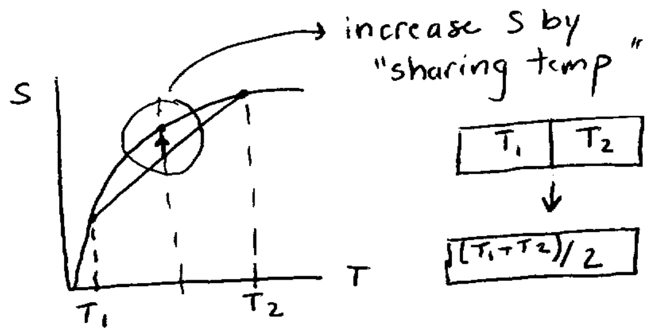
$\bar{q} \to$ heat flux $\left[\frac{J}{m^2\, s} = \frac{W}{m^s}\right]$
$-k \to$ thermal conductivity $\left[\frac{W}{m\, K}\right]$
$\overline{\nabla T} \to$ temperature gradient $\left[\frac{K}{m}\right]$

Compare:

Fourier's Law	**Fick's 1st Law**
$\bar{q} = -k\overline{\nabla T}$	$\bar{J} = -D\overline{\nabla c}$

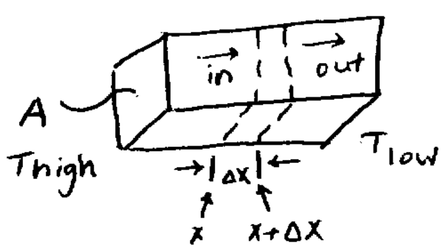
Why does heat behave this way?

Energy minimization $\Rightarrow$ Entropy maximization



Heat Conduction Equation:

First think in 1-D



Heat balance in a small element:

$$\underbrace{\text{heat in}}_{A\cdot q_{\text{in}}} - \underbrace{\text{heat out}}_{A\cdot q_{\text{out}}}\ \underbrace{(+\text{ heat generation})}_{\text{chemical reaction, resistance}} = \underbrace{\text{heat accumulation}}_{V\cdot \frac{\partial H}{\partial T}}$$

$$A\, q_{\text{in}} - A\, q_{\text{out}} = V\frac{\partial H}{\partial T}$$

$$q_{\text{in}} - q_{\text{out}} = \Delta x \frac{\partial H}{\partial T}$$
$$q|_x - q|_{x+\Delta x} = \Delta x \frac{\partial H}{\partial T}$$
$$-k\frac{\partial T}{\partial x}|_x - \frac{\partial T}{\partial x}|_{x+\Delta x} = \Delta x \frac{\partial H}{\partial T}$$
$$\frac{\partial H}{\partial T} = \frac{k}{\Delta x}\left(\frac{\partial T}{\partial x}|_{x+\Delta x} - \frac{\partial T}{\partial x}|_x\right)$$
$$= \frac{k}{\Delta x}\Delta\left(\frac{\partial T}{\partial x}\right)$$
$$= k\frac{\partial(\frac{\partial T}{\partial x})}{\partial x}$$
$$\boxed{\frac{\partial H}{\partial T} = k\frac{\partial^2 T}{\partial x^2}}$$

How does H relate to T?

$$\Delta H = \Delta T\, c_p\, \rho$$ where c_p is heat capacity and ρ is density

$$\frac{\partial H}{\partial t} = \rho\, c_p \frac{\partial T}{\partial t}$$
$$k\frac{\partial^2 T}{\partial x^2} = \rho\, c_p \frac{\partial T}{\partial t}$$
$$\frac{k}{\rho\, c_p}\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$
$$\alpha\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$
$$\boxed{\frac{\partial T}{\partial t} = \alpha\,\frac{\partial^2 T}{\partial x^2}}$$

α is the **Thermal Diffusivity**

$$\alpha = \frac{k\left[\frac{W}{m\,K}\right]}{\rho\left[\frac{kg\,K}{J}\right]\ c_p\left[\frac{m^3}{kg}\right]}$$
$$\boxed{\alpha = \frac{k}{\rho\, c_p}\ \left[\frac{m^2}{s}\right]}$$

The values of k, c_p and ρ for any material can be looked up in tables and do not need to be experimentally determined. Therefore α is a **materials property**.

Compare:

Heat Conduction Equation

$$\frac{\partial T}{\partial t} = \underbrace{\alpha}_{\downarrow} \frac{\partial^2 T}{\partial x^2}$$

thermal diffusivity $\left[\frac{m^2}{s}\right]$

Fick's 2nd Law

$$\frac{\partial c}{\partial t} = \underbrace{D}_{\downarrow} \frac{\partial^2 c}{\partial x^2}$$

diffusivity $\left[\frac{m^2}{s}\right]$

Topic for Future Discussion:

In 3-D ...

$$\rho\, c_p \frac{\partial T}{\partial t} = \nabla \cdot k \nabla T$$

$$\boxed{\frac{\partial T}{\partial t} = \alpha \nabla^2 T}$$

** assuming k is constant with respect to the derivative

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