



LABORATORY EXPERIMENT 2

Origins of Elasticity and Plasticity : The Bubble Raft

PreLab Questions:

1. Explain the mathematical form of a typical interatomic potential and the molecular origins of each term.
2. Explain what a bubble raft is and how it will be used to study elasticity and plasticity.

I. Introduction

The elastic (reversible) and plastic (irreversible) behavior of materials can be described by mechanical properties such as elastic moduli (E) and yield stress (σ_y). These mechanical properties quantify the average behavior of materials that are idealized as semi-infinite blocks with no underlying microstructure, or a *continuum*. However, elastic and plastic properties clearly derive from the arrangement and interaction of atoms and/or molecules that comprise the material. If you deform a material there is a change in the internal energy between atoms, the interatomic potential $U(r)$, where r is the distance between atom centers. The interatomic potential is the summation of attractive and repulsive forces between atoms and characterized by the Lennard-Jones Equation:

$$U(r) = \frac{-A}{r^m} + \frac{B}{r^n}$$

The force felt between atoms is related to the derivative of the interatomic potential and the elastic modulus is related to the second derivative of the interatomic potential.

In this laboratory experiment, we will model and measure the *atomistic and molecular origins* of elasticity and plasticity. By understanding the fundamental mechanisms of deformation, we will be able to predict continuum level mechanical properties, and to suggest atomistic changes to alter these mechanical properties.

In 1947, years before the transmission electron microscope had been developed to observe materials at atomic length scales, Prof. W. Bragg of the Cavendish Laboratory in Cambridge University, UK, conceived of modeling metal crystals via rafts of bubbles. Legend has it that this idea occurred to him while pouring oil into his lawn mower, when he noticed that the bubbles formed in this viscous solution easily formed closely packed rafts that resembled the close-packed {111} plane in crystals. He and his graduate students created rafts in glycerine/soap solutions, simulating defect-free metal crystals, and measured the interbubble potential and elastic and plastic properties of this model metal. It turns out that small bubbles floating on a soap solutions have the same forces between them as atoms in a metal and the interbubble potential is equivalent to the interatomic potential. Having characterized the atomistic and continuum level behavior of this model metal, Bragg et al. later used the bubble raft to consider how defects impact mechanical properties of crystals¹.

¹ Bragg, L., & Nye J.F., (1947). **A Dynamical Model of a Crystal Structure**. *Proc. R. Soc. Lond. Ser. A*, 190 (1023), 474-481.

II. Objectives

The objectives of this experiment are to:

1. Create a perfect single crystal model via a soap bubble raft and an alternative bubble raft.
2. Determine how solution density and internal bubble pressure affect the energetics (interbubble potential) of the model crystal.
3. Compare the atomistic and continuum interpretations of elastic and plastic behavior in the model crystal.

4. Measure and compare the continuum level elastic and plastic properties of the model bubble and the alternative bubble raft.

Equation Sheet for 3.032 Lab 2: Origins of Elasticity and Plasticity

Variables:

T	= surface tension (0.023 N/m)
$\rho_{solution}$	= density of surrounding fluid
g	= gravitational constant
R	= bubble radius
r	= distance between bubble centers

The Bubbles:

In order to simplify the final expression we introduce **Laplace's Constant**, α

$$\alpha^2 = \frac{T}{\rho_{solution} g R}$$

In order for bubbles to behave like atoms in a crystal, internal bubble pressure (P_{int}) must be greater than the hydrostatic pressure acting on the bubble (P_{hydro}).

$$P_{int} > P_{hydro} \quad (1)$$

Where

$$P_{int} = \frac{2T}{R} \quad (2)$$

and

$$P_{hydro} = \rho_{solution} g R \quad (3)$$

Substituting equation (2) and (3) into equation (1) gives

$$\frac{2T}{R} > \rho_{solution} g R$$

or

$$\alpha = \frac{R}{a} < \sqrt{2} \quad (4)$$

Bubble Potentials:

The **attractive potential** between soap bubbles of the same size is

$$U_{attr}(\rho) = -B \cdot Z = -\pi R^4 \rho_{solution} g \left(\frac{\beta}{\alpha} \right)^2 AK_0[\alpha\rho] \quad (5)$$

The **repulsive potential** between soap bubbles of the same size when in contact is

$$U_{rep}(\rho) = \pi R^4 \rho_{solution} g \frac{(2-\rho)^2}{\alpha^2} \quad (6)$$

Combining equations (5) and (6) to get the full form of the **bubble potential**

$$U(\rho) = -\pi R^4 \rho_{solution} g \left(\frac{\beta}{\alpha} \right)^2 AK_0[\alpha\rho] + \begin{cases} \pi R^4 \rho_{solution} g \frac{(2-\rho)^2}{\alpha^2} & \rho \leq 2 \\ 0 & \rho \geq 2 \end{cases} \quad (7)$$

Where

B = buoyancy force

Z = height of solution surface above that at infinity around the second bubble

$\alpha = R/a$, ratio of bubble radius to Laplace constant

$\beta = b/R$ = dimensionless radius of ring contact

$\rho = r/R$ = ratio of distance between bubbles to bubble radius

A = constants based on boundary conditions

$K_0[x]$ = zeroth order modified Bessel function of the second kind.

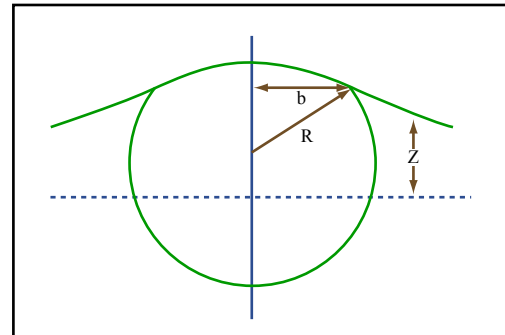


Figure by MIT OpenCourseWare.

Figure 1: Effect of bubble on the curvature of the soap solution

Hint: In Mathematica use “BesselK[0,x]” or the Bessel function can be approximated as

$$K_0(x) \approx \frac{e^{-x}}{\sqrt{\frac{2x}{\pi}}} \quad \text{for } x \gg 0$$

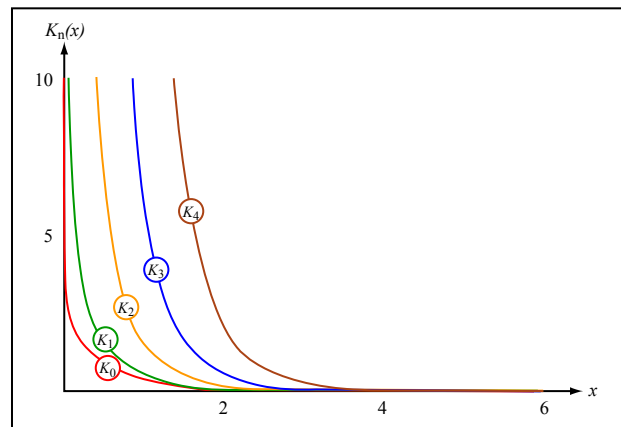


Figure by MIT OpenCourseWare.

Figure 2: Modified Bessel functions of the second kind

α	β	A
0	0	0
0.05	0.056870	0.0016178
0.10	0.115020	0.0066280
0.15	0.172800	0.0150075
0.20	0.230270	0.0267946
0.25	0.287280	0.0420500
0.30	0.343550	0.0608386
0.35	0.398760	0.0832360
0.40	0.452560	0.1093190
0.45	0.504570	0.1391430
0.50	0.554440	0.1727460
0.55	0.601870	0.2101400
0.60	0.646600	0.2512930
0.65	0.688430	0.2961180
0.70	0.727240	0.3444930
0.75	0.762980	0.3962520
0.80	0.795650	0.4511810
0.85	0.825310	0.5093600
0.90	0.852040	0.5695130
0.95	0.875980	0.6323320
1.00	0.897270	0.6971610
1.05	0.916060	0.7636470
1.10	0.932500	0.8314080
1.15	0.946770	0.9001340
1.20	0.959000	0.9693830
1.25	0.969350	1.0388100
1.30	0.977970	1.1080700
1.35	0.984970	1.1767200
1.40	0.990500	1.2449000
1.45	0.994660	1.3109600
1.50	0.997560	1.3758200

Figure 3: β and A for different values of α

Pendulum Device:

Force (F) on pendulum from mass (m)

$$F = \frac{mgx}{h}$$

where

x = distance out on lever arm

h = height of lever arm

Note: This is used to calibrate pendulum

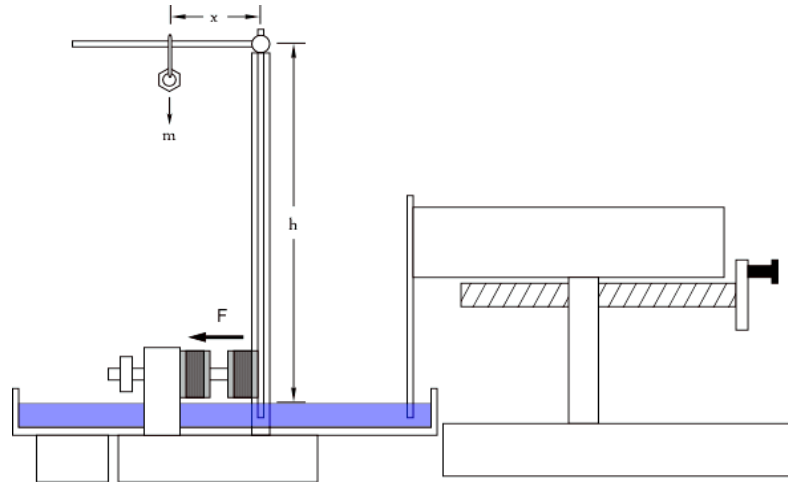


Figure 4: Pendulum

Stress (σ) on bubble raft

$$\sigma = F/w$$

where

F = force exerted on pendulum by the bubbles

w = width of bubble raft against flat

Note: Because the raft is 2D, stress here will be in N/m instead of the usual N/m²