24.910 Topics in Linguistic Theory: Propositional Attitudes
Spring 2009

1. From Last Time: Introducing Intensional Semantics [Summary]

We can set up a lot of the mechanics of an intensional semantics using a simplistic but useful example of a fictional world.

1.1. Version I: "The world of Sherlock Holmes"

❖ **Informal idea**

The modifier signals that the sentence is to be evaluated at a particular world – say, w_9 (the world of Sherlock Holmes) – rather than at the actual world

❖ **More formally:**

(1) $[\![\text{In the world of Sherlock Holmes, } \phi]\!]^w = [\![\phi]\!]^{w9}$

[i.e., *in the world of Sherlock Holmes, ϕ* is true in any world w iff ϕ is true in w_9.]

(2) $[\![\text{In the world of Sherlock Holmes}]\!]^w = \lambda p_{<s,t>} \,.\, p(w_9)$

1.2. Version II: The world of Sherlock Holmes as presented by Sir Arthur Conan Doyle in world w

➢ A problem with the idea in 1.1:

It's a contingent fact that Sir Arthur Conan Doyle wrote the Sherlock Holmes stories the way he did. He might have (in some other world) set things up so that Sherlock Holmes lived on Abbey Rd (and no detective lived on Baker St.)

(3) $[\![\text{In the world of Sherlock Holmes, } \phi]\!]^w = [\![\phi]\!]^{sher(w)}$

(4) $[\![\text{In the world of Sherlock Holmes, } \phi]\!]^w = \lambda p_{<s,t>} \,.\, p(sher(w))$

$= \lambda p_{<s,t>}$. the world w' as it is described in the Sherlock Holmes stories as written in w is such that p(w') = 1

1.3. Version III: The set of worlds compatible with what is presented by Sir Arthur Conan Doyle in world w

➢ A problem with the idea in 1.2: Sir Arthur Conan Doyle didn't actually make every aspect of his fictional world explicit. For example, we don't know whether Holmes had an odd or even number of hairs on his head the day he met Watson (and in some sense Sir Arthur Conan Doyle doesn't know either!)

(5) $[\![\text{In the world of Sherlock Holmes, } \phi]\!]^w = 1 \text{ iff } \forall w' \in sher(w), [\![\phi]\!]^{w'} = 1$

(6) $[\![\text{In the world of Sherlock Holmes, } \phi]\!]^w = \lambda p_{<s,t>} \,.\, \text{for all } w' \in sher(w), p(w') = 1$

2. Semantics of Attitude Predicates

2.1. The Idea

Recall: the function **sher**:

- sher(w) = {w': w' is compatible with the world depicted in the Sherlock Holmes stories, as written in w}

We can define a similar function for a person's beliefs (and other attitudes):

(7) $Bel_{x,w}$ = {w': w' is compatible with what x believes in w}

[Note: We haven't said much of anything about what it means to be "compatible with what x believes" – more on this later.]

2.2. Lexical Entries

Using functions of this kind:

(8) $[\![\text{x believes } \phi]\!]^w = 1 \text{ iff } \forall w' \in Bel_{x,w} : [\![\phi]\!]^{w'} = 1$

Breaking it down:

(9) $[\![\text{believe}]\!]^w = \lambda p_{<s,t>} . \lambda x_e . \forall w' \underline{\in Bel_{x,w}} : p(w') = 1$
$= \lambda p_{<s,t>} . \lambda x_e . \forall w'$: <u>w' is compatible with what x believes in w</u>: p(w') = 1

(10) $[\![\text{think}]\!]^w = [\![\text{believe}]\!]^w$

Of course, we can do something parallel for other attitudes:

(11) $[\![\text{know}]\!]^w = \lambda p_{<s,t>} . \lambda x_e . \forall w'$: <u>w' is compatible with what x knows in w</u>: p(w') = 1

(12) $[\![\text{suspect}]\!]^w$
$= \lambda p_{<s,t>} . \lambda x_e . \forall w'$: <u>w' is compatible with what x suspects in w</u>: p(w') = 1

(13) $[\![\text{imagine}]\!]^w$
$= \lambda p_{<s,t>} . \lambda x_e . \forall w'$: <u>w' is compatible with what x imagines in w</u>: p(w') = 1

(14) $[\![\text{want}]\!]^w = \lambda p_{<s,t>} . \lambda x_e . \forall w'$: <u>w' is compatible with what x wants in w</u>: p(w') = 1

Obviously this doesn't tell us much about the <u>lexical</u> semantics …

2.3. Getting the Types Right

The attitude predicates above have semantic type < <s,t>, <e,t> >

If we write the semantics for a sentence like this:

- $[\![\text{It's raining}]\!]^w$ = 1 iff it's raining in w

Then this is technically the wrong type (type t, rather than type <s,t>)

To fix this [one simple option]:

- Stipulate that expressions can freely shift to their <u>intensions</u>:
- The intension of α is the function $\boldsymbol{\lambda w' . [\![\alpha]\!]^{w'}}$

2.4. *Exercises*

Compute the truth conditions of (15) at a particular world w_1:

(15) Sue thinks that it's raining.

- Note: *(that) it's raining* will have to shift to its intension:
  Instead of **$[\![$it's raining$]\!]^w$ = 1 iff it's raining in w**
  We use: **λw" . it's raining in** w"
- $[\![(15)]\!]^{w1} = [\![\text{think}]\!]^{w1}$ ([λw' . $[\![$it's raining$]\!]^{w'}$]) ($[\![\text{Sue}]\!]^{w1}$)

  = [λ$p_{<s,t>}$. λx_e . ∀w': <u>w' is compatible with what x believes in w</u>: p(w') = 1] ([λw" . it's raining in w"]) (Sue)

  = [λx_e . ∀w': <u>w' is compatible with what x believes in w</u>: **[λw" . it's raining in w"]** (w') = 1] (Sue)

  = [λx_e . ∀w': <u>w' is compatible with what x believes in w</u>: **it's raining in w'**] (Sue)

  = 1 iff ∀w': <u>w' is compatible with what **Sue** believes in w</u>: it's raining in w'

2.5. *Attitude predicates in set talk*

Lexical entry for *believe* in set talk:

(16) $[\![\text{believe}]\!]^w = \lambda p_{<s,t>} . \lambda x_e . Bel_{x,w} \subseteq p$

Show what goes wrong if we use the wrong set relation (exercise 2.1 in von Fintel & Heim):

❖ **WRONG LEXICAL ENTRY I:**

(17) $\mathbf{[\![\text{believe}]\!]^w = \lambda p_{<s,t>} . \lambda x_e . p = Bel_{x,w}}$

Compute truth conditions of (15) at world w_1 using meaning in (17):

- $[\![\text{think}]\!]^{w1} = [\![\text{believe}]\!]^{w1} = \lambda p_{<s,t>} . \lambda x_e . p = Bel_{x,w1}$
- $[\![$Sue thinks that it's raining$]\!]^{w1}$ = [λ$p_{<s,t>}$. λx_e . p = $Bel_{x,w1}$] ([λw" . it's raining in w"]) (Sue)

  = [λx_e . **[λw" . it's raining in w"]** = $Bel_{x,w1}$] (Sue)

Translate into set talk…

= [λx_e . **{w": it's raining in w"}** = $Bel_{x,w1}$] (Sue)

= 1 iff {w": it's raining in w"} = $Bel_{Sue,w1}$

= 1 iff the set of worlds compatible with Sue's beliefs consists of all and only those worlds where it's raining

→ Requires that any situation where it's raining is a possibility as far as Sue is concerned (including ones where, for example, pink unicorns have taken over the earth). → Too strong!

❖ **WRONG LEXICAL ENTRY II:**

(18) $[\![\textbf{believe}]\!]^w = \lambda p_{<s,t>} . \lambda x_e . p \cap Bel_{x,w} \neq \varnothing$

Compute truth conditions of (15) at world w_1 using meaning in (18):

- $[\![\text{think}]\!]^{w1} = [\![\text{believe}]\!]^{w1} = \lambda p_{<s,t>} . \lambda x_e . p \cap Bel_{x,w1} \neq \varnothing$
- $[\![\text{Sue thinks that it's raining}]\!]^{w1} = [\lambda p_{<s,t>} . \lambda x_e . p \cap Bel_{x,w} \neq \varnothing]$ ($[\lambda w'' .$ it's raining in w'']) (Sue)

  = $[\lambda x_e$. **[λw'' . it's raining in w'']** $\cap Bel_{x,w} \neq \varnothing]$ (Sue)

Translate into set talk…

= $[\lambda x_e$. **{w'': it's raining in w''}** $\cap Bel_{x,w} \neq \varnothing]$ (Sue)

= 1 iff **{w'': it's raining in w''}** $\cap Bel_{Sue,w} \neq \varnothing$

= 1 iff there are some worlds compatible with Sue's beliefs such that it's raining

→ Only requires that Sue not have a definite belief that it's NOT raining – for example, she could have no idea whether it's raining or not. → Too weak!

3. Accessibility Relations

Q. what does it mean to be "compatible with" a person's knowledge, beliefs, etc.?

We won't really answer this, but we can say a little bit more about knowledge, belief, etc. [and thus about the lexical semantics of *know*, *believe*, etc.]

One thing that helps: hold the subject and type of attitude constant, and consider mental states as relations between worlds:

Another notation:

(19) **w R_x^{Bel} w'**
= w' is compatible with x's beliefs in w

When the subject and attitude type are understood, we might write **wRw'**.

Some terminology:

- **R**'s of this type are called <u>accessibility relations</u>.
- **wRw'** can be read as **w' is accessible to w** / (sometimes) **w sees w'**

We can say something more about propositional attitudes by talking about the properties of these various accessibility relations

3.1.1. Some properties of relations

❖ **Reflexivity**

R is <u>reflexive</u> iff for all x in the domain of R, **xRx**

❖ **Transitivity**

R is <u>transitive</u> iff whenever **xRy** and **yRz**, it's also the case that **xRz**

❖ **Symmetry**

R is <u>symmetric</u> iff whenever **xRy**, it's also the case that **yRx**

3.1.2. Accessibility Relations for *know*

- **Reflexive**
  (This reflects the intuition that you can only <u>know</u> things that are true)
- **Transitive?**

  Maybe… This depends on whether we want to assume that if you know something, then you know that you know it
- **Symmetric?**

  Probably not … This depends on whether we want to assume that if something happens to be true in the actual world, then you <u>know</u> that it's compatible with your knowledge

3.1.3. Accessibility Relations for *believe*

- **NOT reflexive**
  (because you can believe things that are false)
- **Transitive**
  If you believe something, then you believe that you believe it
- **NOT Symmetric**

  Something can be the case in the actual world which you do not believe to be compatible with your beliefs

3.1.4. Accessibility Relations for *want*

- **NOT reflexive**
  (because you can want things to be the case that are not the case)
- **NOT Transitive**
  (because presumably you can want something without <u>wanting</u> to want it)
- **NOT Symmetric**

Something can be the case in the actual world which is not compatible with what you want yourself to want

[Obviously there's a lot more to say about these relations than these three properties, but this gives us a framework]

3.2. Selected Exercises

[Possibly work through in class depending on time]

- **Exercise 2.3 (p. 20)**

- **Think about the accessibility relations involved in the following predicates:**
  - *suspect*
  - *imagine*
  - [More???]