Subject 24.242. Logic II. Sample problems from the second homework, due March 4.

A *register machine* consists of an infinite number of memory locations, named Register 1, Register 2, Register 3, and so on, each of which is capable of holding a natural number. A *register program* is a finite numbered list of instructions, which take the following five forms:

Add 1 to the number in Register i.
Subtract 1 from the number in Register j, unless that number is already 0.
If the number in Register k is 0, go to instruction m
Go to instruction n.
STOP.

A computation starts at the first instruction, and proceeds from an instruction to the next, unless instructed otherwise. To calculate an n-ary partial function, begin with the inputs in Registers 1 through n, and with zero in all the other registers. If the computation eventually reaches the STOP instruction, the computation halts, and the number in Register 1 is the output. If the computation never reaches the STOP instruction, the function is undefined for that input. For example, the following program computes the successor function:

1. Add 1 to Register 1.
2. Stop.

The following program computes the characteristic function of the identity relation, the binary function that yields output 1 if $x = y$ and 0 if $x \neq y$:

1. If the number in Register 1 is 0, go to instruction 6.
2. If the number in Register 2 is 0, go to instruction 10.
3. Subtract 1 from the number in Register 1, unless that number is already 0.
4. Subtract 1 from the number in Register 2, unless that number is already 0.
5. Go to instruction 1.
6. If the number in Register 2 is 0, go to instruction 8.
7. STOP.
8. Add 1 to the number in Register 1.
9. STOP.
10. Subtract 1 from the number in Register 1, unless that number is already 0.
11. If the number in Register 1 is 0, go to instruction 9.
12. Go to instruction 10.

1. Write a register program that calculates $(x + y)$.

2. Show that a set is Δ if and only if its characteristic function is Σ. (The *characteristic function* χ_S of a set S is given by stipulating that $\chi_S(n) = 1$ if $n \in S$, and it's equal to 0 if $n \notin S$.)