10.2E

Question 6

Part (c)

1	$(\exists x)(\exists y)(Fxy \lor Fyx)$	A
2	$(\exists y)(Fay \lor Fya)$	A/∃E
3	$Fab \lor Fba$	A/∃E
4	Fab	A/∨E
5	$(\exists y)Fay$	4, ∃I
6	$(\exists x)(\exists y)Fxy$	5, ∃I
7	Fba	A/∨E
8	$(\exists y)Fby$	7, ∃I
9	$(\exists x)(\exists y)Fxy$	8, ∃I
10	$(\exists x)(\exists y)Fxy$	3, 4-6, 7-9, ∨E
11	$(\exists x)(\exists y)Fxy$	2, 3-10, ∃E
12	$(\exists x)(\exists y)Fxy$	1, 2-11, ∃E

Question 7

Part (d)

1	$(\exists x)Fxx$	A/⊃I
2	Faa	A/∃E
3	$(\exists y)Fay$	2, ∃I
4	$(\exists x)(\exists y)Fxy$	3, ∃I
5	$(\exists x)(\exists y)Fxy$	1, 2-4, ∃E
6	$(\exists x)Fxx \supset (\exists x)(\exists y)Fxy$	1-5, ⊃I

Part (j)

1	$(\exists x)(\forall y)Hxy$	A/⊃I
2	$(\forall y)Hay$	A/∃E
3	Hab	2,∀E
4	$(\exists x)Hxb$	3, ∃I
5	$(\forall y)(\exists x)Hxy$	4, ∀I
6	$(\forall y)(\exists x)Hxy$	1, 2-5, ∃E
7	$(\exists x)(\forall y)Hxy \supset (\forall y)(\exists x)Hxy$	1-6, ⊃I

Question 8

Part (c)

First, I derive '$(\exists x)(\forall y)(Fx \supset Hxy)$' from '$(\exists x)[Fx \supset (\forall y)Hxy]$'.

1	$(\exists x)[Fx \supset (\forall y)Hxy]$	A
2	$Fa \supset (\forall y)Hay$	A/∃E
3	Fa	A/⊃I
4	$(\forall y)Hay$	2, 3, ⊃E
5	Hab	4, ∀E
6	$Fa \supset Hab$	3-5, ⊃I
7	$(\forall y)(Fa \supset Hay)$	6, ∀I
8	$(\exists x)(\forall y)(Fx \supset Hxy)$	7, ∃I
9	$(\exists x)(\forall y)(Fx \supset Hxy)$	1, 2-8, ∃E

Now I derive '$(\exists x)[Fx \supset (\forall y)Hxy]$' from '$(\exists x)(\forall y)(Fx \supset Hxy)$' .

1	$(\exists x)(\forall y)(Fx \supset Hxy)$	A
2	$\quad(\forall y)(Fa \supset Hay)$	A/∃E
3	$\quad Fa \supset Hab$	2, ∀E
4	$\qquad Fa$	A/⊃I
5	$\qquad Hab$	3, 4, ⊃E
6	$\qquad(\forall y)Hay$	5, ∀I
7	$\quad Fa \supset (\forall y)Hay$	4-6, ⊃I
8	$\quad(\exists x)[Fx \supset (\forall y)Hxy]$	7, ∃I
9	$(\exists x)[Fx \supset (\forall y)Hxy]$	1, 2-8, ∃E

The Final Derivation

This is a little tricky.

I wouldn't be surprised if there is a more efficient way to do this then the one below.

1	$(\forall x)(\exists y)(\sim y = a \,\&\, (\forall w)(Rwy \equiv x = w))$	A
2	$\sim (\exists x)(\exists y)(\exists z)(\sim x = y \,\&\, (\sim x = z \,\&\, \sim y = z))$	A
3	$(\exists y)(\sim y = a \,\&\, (\forall w)(Rwy \equiv a = w))$	1, ∀E
4	$(\exists y)(\sim y = a \,\&\, (\forall w)(Rwy \equiv b = w))$	1, ∀E
5	$\sim b = a \,\&\, (\forall w)(Rwb \equiv a = w))$	A/∃E
6	$\sim b = a$	5, &E
7	$(\forall w)(Rwb \equiv a = w)$	5, &E
8	$\sim c = a \,\&\, (\forall w)(Rwc \equiv b = w)$	A/∃E
9	$\sim c = a$	8, &E
10	$(\forall w)(Rwc \equiv b = c)$	8, &E
11	$b = c$	A/∼I
12	$Rcc \equiv b = c$	10, ∀E
13	Rcc	11, 12, ≡E
14	Rbb	11, 13, =E
15	$Rbb \equiv a = b$	7, ∀E
16	$a = b$	14, 15, ≡E
17	$a = a$	16, 16 =E
18	$\sim a = a$	6, 16, =E
19	$\sim b = c$	11-18, ∼I
20	$\sim b = a \,\&\, \sim c = a$	6, 9 &I
21	$\sim b = c \,\&\, (\sim b = a \,\&\, \sim c = a)$	19, 20, &I
22	$(\exists z)(\sim b = c \,\&\, (\sim b = z \,\&\, \sim c = z))$	21, ∃I
23	$(\exists y)(\exists z)(\sim b = y \,\&\, (\sim b = z \,\&\, \sim y = z))$	22, ∃I
24	$(\exists x)(\exists y)(\exists z)(\sim x = y \,\&\, (\sim x = z \,\&\, \sim y = z))$	23, ∃I
25	$(\exists x)(\exists y)(\exists z)(\sim x = y \,\&\, (\sim x = z \,\&\, \sim y = z))$	4, 8-24, ∃E
26	$(\exists x)(\exists y)(\exists z)(\sim x = y \,\&\, (\sim x = z \,\&\, \sim y = z))$	3, 5-25, ∃E
27	$\sim (\exists x)(\exists y)(\exists z)(\sim x = y \,\&\, (\sim x = z \,\&\, \sim y = z))$	2, R

24.241 Logic I
Fall 2009