

The Cantorian Theory of Size

1 Basic Definitions

Cardinality $|A|$ is the size of set A .

Bijection Principle $|A| = |B|$ iff there is a bijection from A to B .

Injection Principle $|A| \leq |B|$ iff there is an injection A to B .

2 Extended Definitions

Notation	How it's defined	Informal notion
$ A = B $	bijection from A to B	just as many members in A as in B
$ A \leq B $	injection from A to B	at most as many members in A as in B
$ A < B $	$ A \leq B $ and $ A \neq B $	fewer members in A than in B
$ A \geq B $	$ B \leq A $	at least as many members in A as in B
$ A > B $	$ A \geq B $ and $ A \neq B $	more members in A than in B

3 Properties of \leq

Reflexivity $|A| \leq |A|$

Anti-symmetry If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$

Transitivity If $|A| \leq |B|$ and $|B| \leq |C|$, then $|A| \leq |C|$

Totality* For any sets A and B , either $|A| \leq |B|$ or $|B| \leq |A|$

*One can only prove Totality if one assumes a controversial set-theoretic axiom: the Axiom of Choice. We'll come across this axiom again. Stay tuned!

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