Wind Turbine Design Optimization

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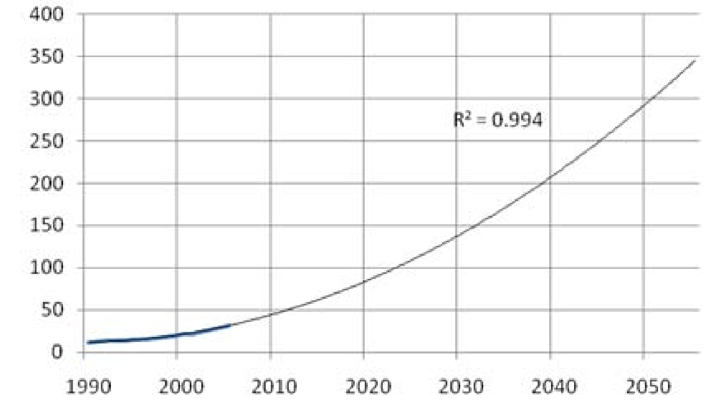
Presentation Outline max(content), min(time)

- Introduction
- Problem Formulation
- Design Vector
- Analysis Methodology & Parameters
- Fidelity & Complexity
- Optimization Methods
- Single Objective Results
- Multi Objective Results
- Conclusions & Future Work

Introduction

Renewable Energy Projection



Source: http://www.paulchefurka.ca/WEAP/WEAP.html

- Focus on renewable energy
- Disciplines involved
  - Aerodynamics
  - Control
  - Structures
  - Acoustics
  - Electrical engineering

(Aerodynamics, Control, Structures:) Considered in this project

- Interactions
  - Control (rotational speed affects aerodynamics)
  - Structures (blade deflection affects aerodynamics)

Problem Formulation

- Objective Function

$$\text{Objective} = J(x, param) = \frac{\text{Expected Power Output}}{\text{Blade Material Volume}} = \frac{P_E(x, param)}{V_{blades}(x, param)}$$

  - Over a range of incoming wind speeds

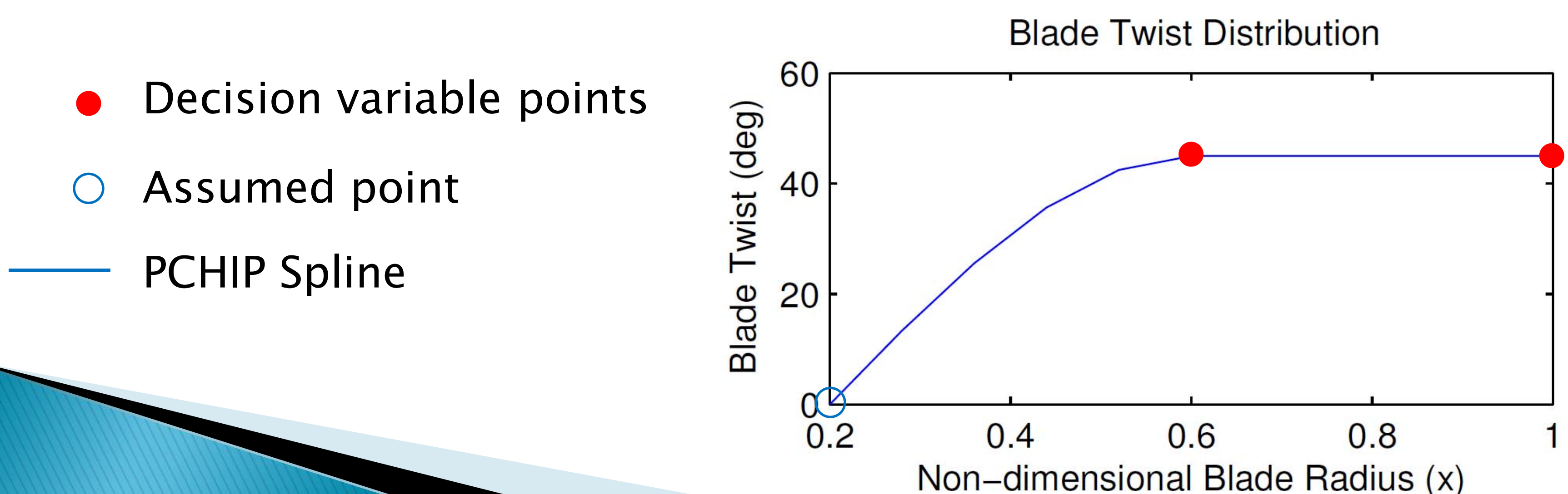
- Penalty Function

$$\xi(x, param) = \sum \left[\rho_{penalty} \left(\left(\sigma^{r}_{max} - \sigma_{allowable} \right)^{+} \right)^{2} \right]$$

Design Vector

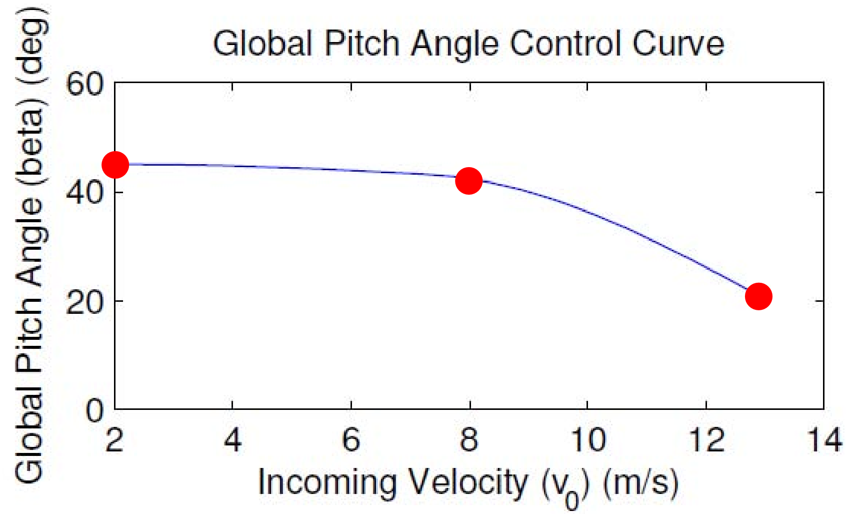
Decision variable	Symbol	Dimension	Lower bnd.	Upper bnd.	Units
Blade radius	R	1	5.00	16.15	meters
Maximum generator torque	$Qmax$	1	1000	20000	Newton-meters
Blade shell thickness	t	1	0.004	0.020	meters
Maximum design wind speed	k	1	0	4	std. dev. above mean
Twist distribution	T	2	0	$\pi/4$	radians
Foil shape distribution	F	1	0	1	non-dimensional
Chord length distribution	C	3	0.1	10	meters
Pitch control curve	β	3	0	$\pi/4$	radians

- Model reduction by Piecewise Cubic Hermite Interpolating Polynomials



Design Vector (beta, k)

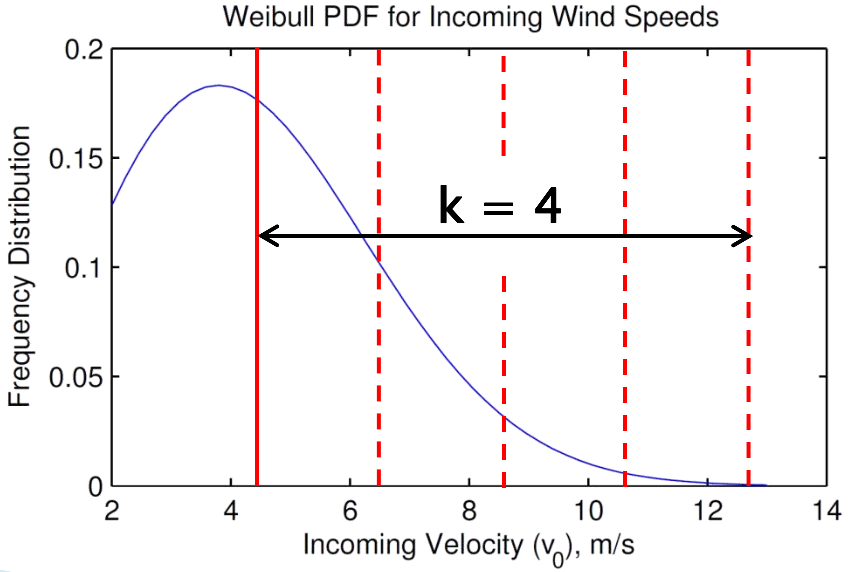
- Decision variable points



<u>Weibull statistics</u>

Mean = 4.43 m/s

Std. Dev. = 2.13 m/s



Analysis Methodology

- N^2 Diagram

Inputs					
x, param	x, param	x, param	x, param	x, param	Output
Wrapper	$c_{weibull}$, $k_{weibull}$				P_E, V_{blades}, $Max(\sigma_{max}(v_0))$
$Max(\sigma_{max}(v_0))$	Expected Power	v_0	ω, v_0	ω, $F_t/\Delta R$, $F_a/\Delta R$	
	ω	Control	ω, v_0		
	Q, P, $F_t/\Delta R$, $F_a/\Delta R$	Q, P, $F_t/\Delta R$, $F_a/\Delta R$	Aero		
	$\sigma_{max}(v_0)$			Structure	

- Aero – Blade Element Momentum Theory
  - Relaxed iteration root-finding
- Expected Power – Simpson's rule integration over Weibull distribution
- Structure – Equivalent beam theory
- Control – Line search convex optimization (fminbnd in MATLAB)

Inter-module optimization

Parameters

Parameter	Symbol	Value	Units
Air density	ρ_{air}	1.225	kg/m^3
Blade material density	ρ_{mat}	2700	kg/m^3
Blade material elasticity	E_{mat}	70e9	Pa
Blade material yield strength	σ_Y	20	MPa
Hub radius/blade radius	h_R	0.2	-
Number of blades	Z	3	-
Cut-in velocity	v_{cutin}	2.0	m/s
Allowable stress/yield stress	$perc_Y$	0.70	-
Weibull distribution scale parameter	$c_{weibull}$	5	-
Weibull distribution shape parameter	$k_{weibull}$	2.2	-
Root foil	-	S814	-
Tip foil	-	S813	-
Cost of blade material	c_1	1000	$\$/m^3$
Penalty parameter	$\rho_{penalty}$	1e-8	\$/Pa
Points in blade discretization	n	11	-
Number of wind speeds in Weibull integration	n_W	7	-

Discretization Parameters (affect fidelity)

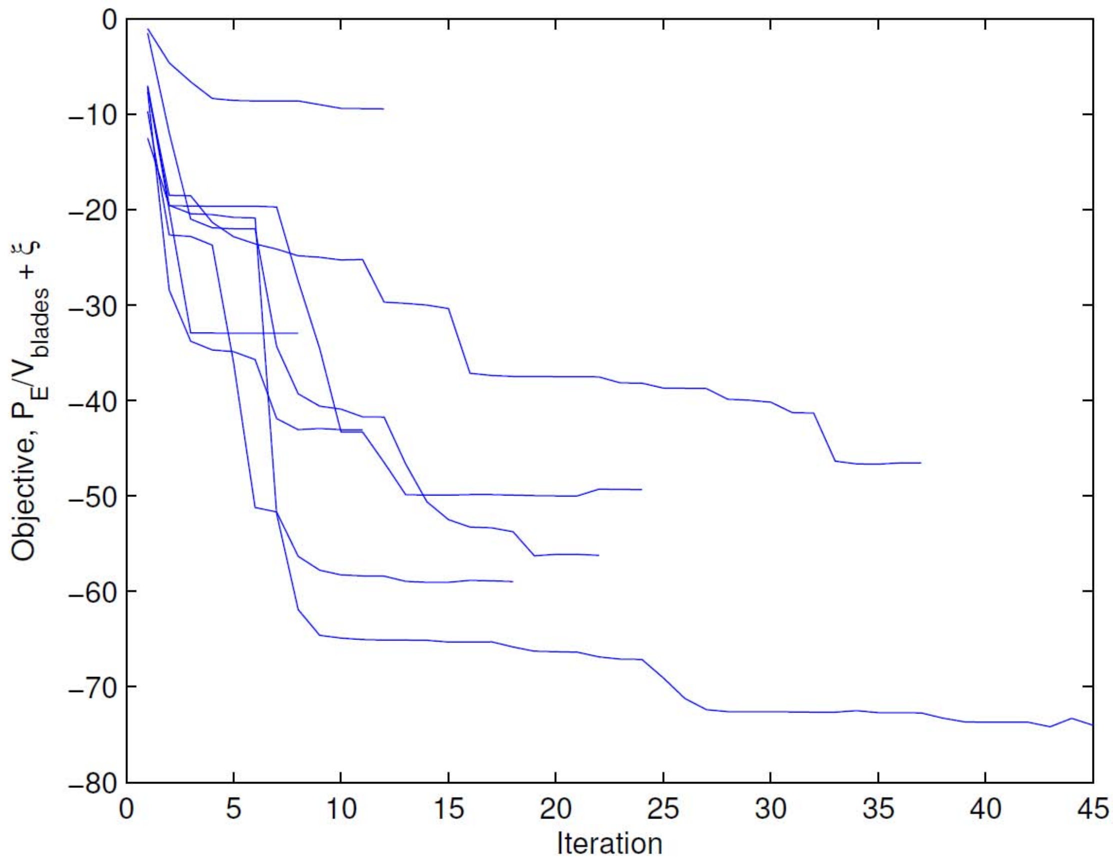
Fidelity/Complexity

- Model order reduction
  - PCHIP
- Analysis methodology low fidelity
  - Quick computation times
- Validation
  - Structures code equates with analytical classical beam theory for cantilevered beam
  - Ran out of time to compare with Qprop / VABS high-fidelity codes
  - Tell us if you know of a wind turbine design to benchmark against

Optimization Methods

- Design of Experiments (DOE)
  - Complex design space (initial idea)
  - Main effects
  - Space-filling starting points for Gradient-based methods
  - Good results, short running times
- Gradient-based methods
  - Continuous design variables with no discontinuities
  - Constraints imposed by square term penalty method
  - Implemented SQP with MATLAB's 'fmincon'
  - Re-scaling Hessian
  - Multi-start (non-convex objective function and feasible space)
  - Good results, long running times
- Heuristic methods
  - Multi-Objective Genetic Algorithm (MOGA)
  - Poor results, long running times

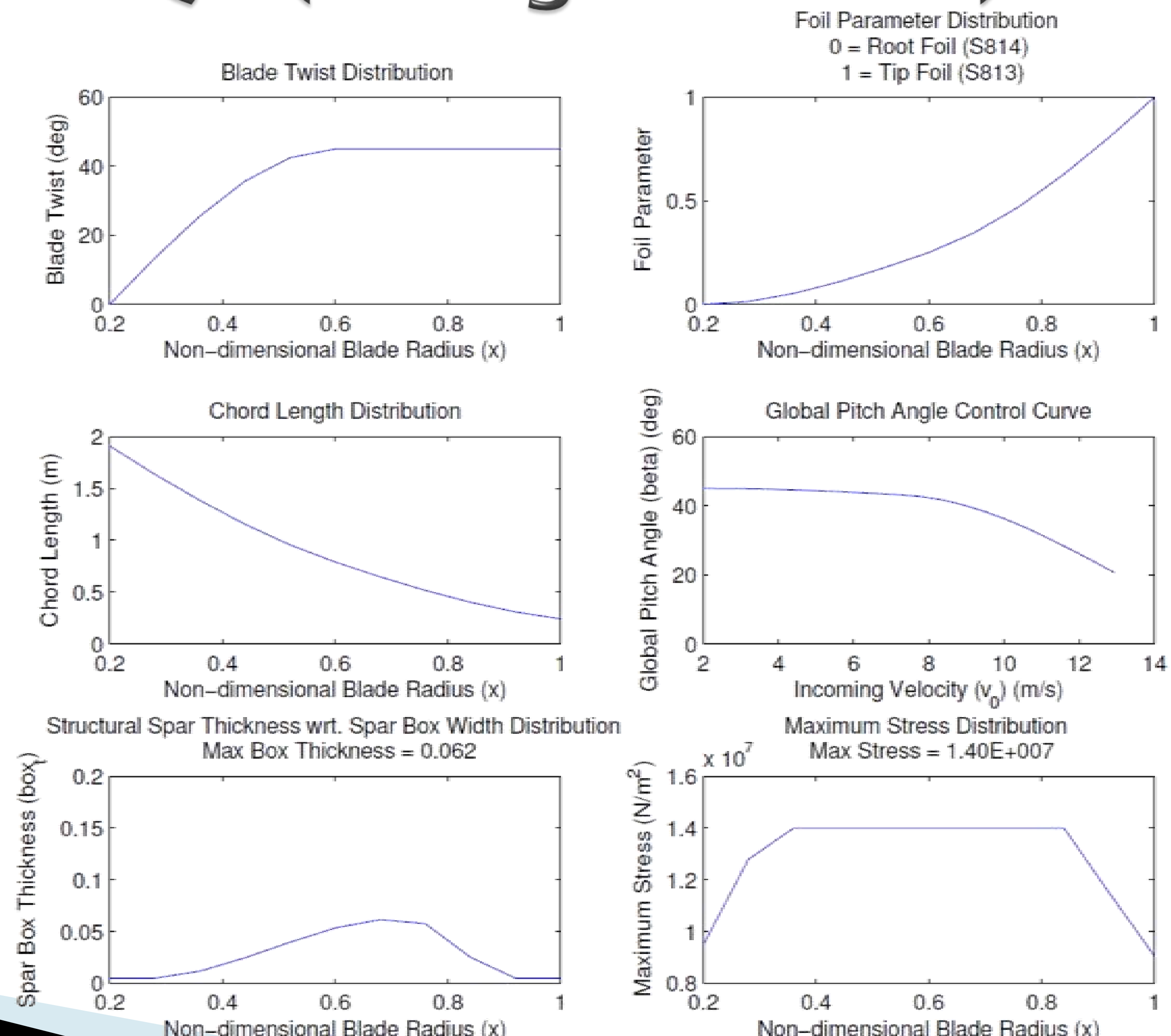
SQP Convergence Sample



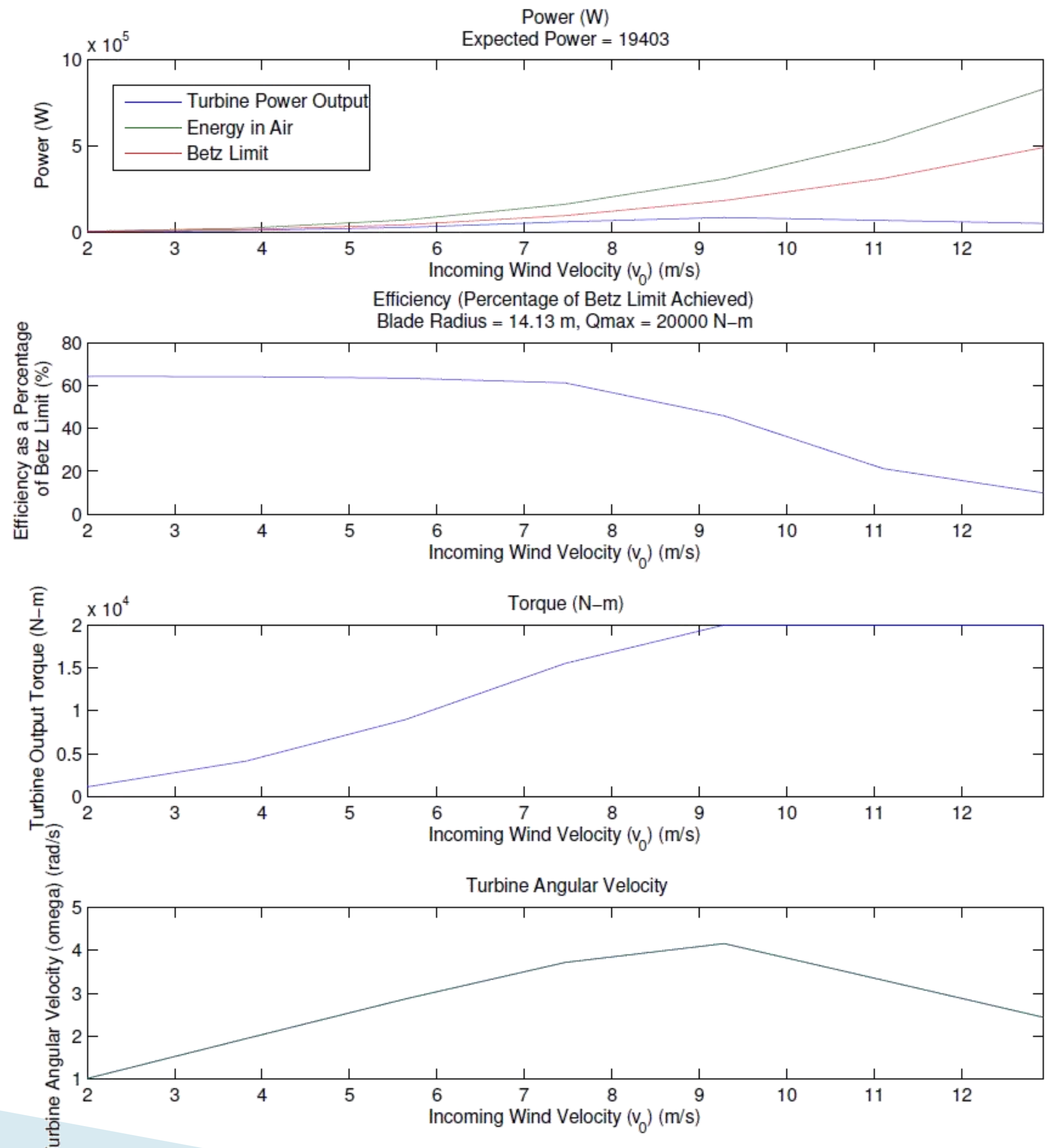
-Varying convergence rates

-Varying solution values → Non-convexity

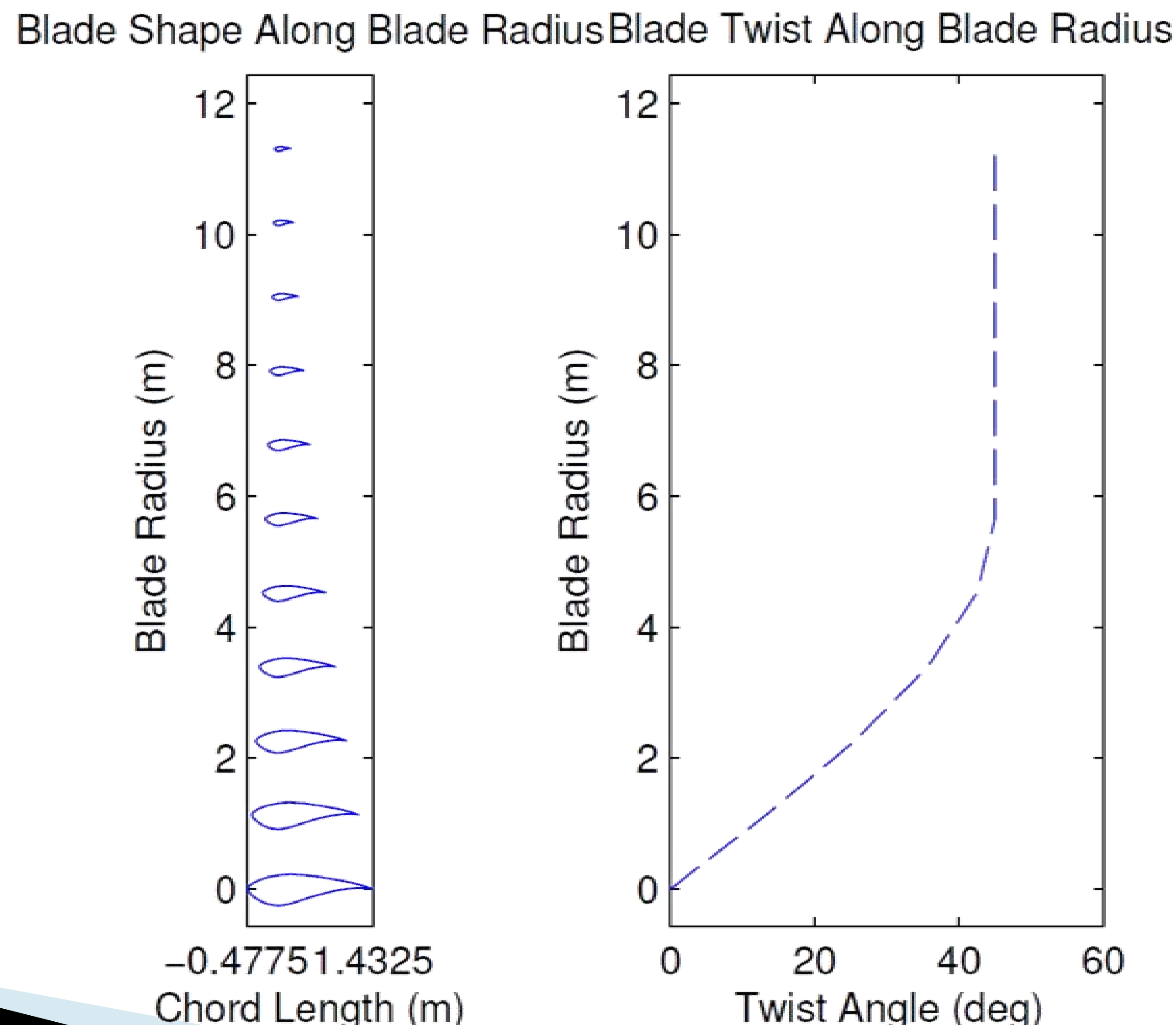
Results – SQP (Design Vector)



Results – SQP (Performance)



Results – SQP (blade shape)



Results – Tight Bounds

Design Variables	Values	Comments		
R	14.13			
Qmax	20000	↑		
t	0.004	↓		
k	4	↑		
T(mid)	0.79	↑	↓	lower bound
T(tip)	0.78	↑		
F	0.25			
C(root)	1.91		↑	upper bound
C(mid)	0.79			
C(tip)	0.24			
beta	0.79	↑		
	0.75			
	0.36162			

Sensitivity Analysis

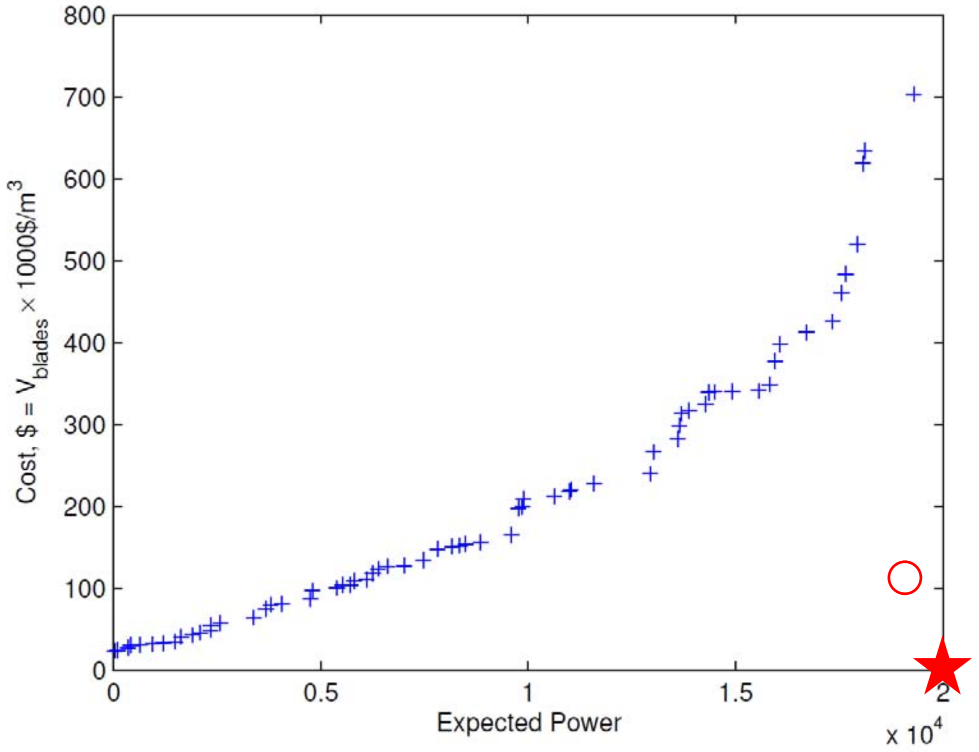
	$\frac{\partial J}{\partial R}$	$\frac{\partial J}{\partial Qmax}$	$\frac{\partial J}{\partial t}$	$\frac{\partial J}{\partial k}$	$\frac{\partial J}{\partial T_{mid}}$	$\frac{\partial J}{\partial T_{tip}}$	$\frac{\partial J}{\partial F_{mid}}$
$J_1 = P_E$	1.510	0.041	0	0.367	0.858	0.2966	-0.007
$J_2 = V_{blades}$	1.0000	0	0.8557	0	0	0	0.0024
$J_3 = \sigma_{max}$	0.0823	-0.0161	0.0434	0.1015	0.1661	0.1856	-0.0005

	$\frac{\partial J}{\partial C_{root}}$	$\frac{\partial J}{\partial C_{mid}}$	$\frac{\partial J}{\partial C_{tip}}$	$\frac{\partial J}{\partial \beta_{cutin}}$	$\frac{\partial J}{\partial \beta_{mean}}$	$\frac{\partial J}{\partial \beta_{cutoff}}$
$J_1 = P_E$	-0.122	0.368	0.150	0.067	1.787	0.045
$J_2 = V_{blades}$	0.5382	0.5638	0.0486	0	0	0
$J_3 = \sigma_{max}$	0.0260	0.0024	-0.0010	-0.0706	0.0791	0.0115

- Sensitivity Analysis (2nd order central difference)
  - Decision variables that are tight on box bounds have directions of improvement without violating feasibility (Qmax increase → P_E/V_{blades} increase, σ_{max} decrease
  - Decision variables that are free on box bounds have no directions of improvement that do not violate constraints (R increase → σ_{max} increase)
- Connection to Lagrange multipliers

Results – MOGA

- Slope of Pareto Front
  - initially benefit from going to higher expected power
  - later stages cost outweighs benefits of increasing expected power
  - optimum somewhere in between
- Utopia point – highest expected power for the lowest cost
- SQP outperforms MOGA
  - Not enough running time
  - Computational expense



○ Best result from SQP

★ Utopia point

Conclusions

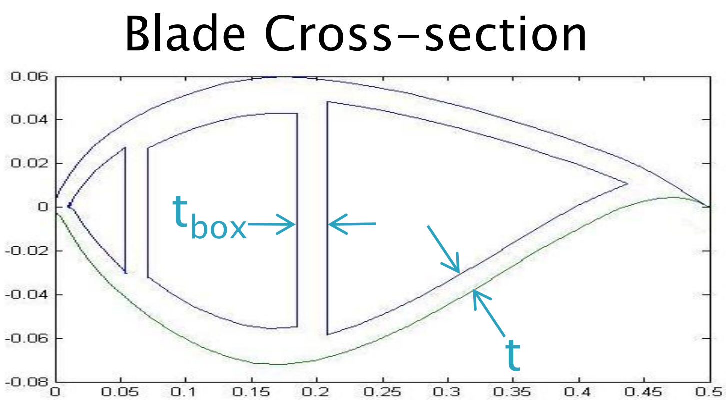
- DOE is powerful and inexpensive
- SQP works great for local optimization
- Heuristic methods may be too expensive

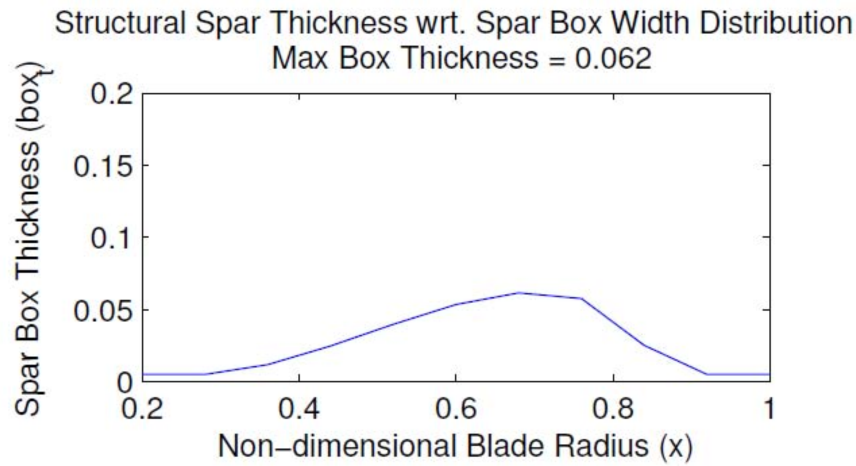
Future Work

- Higher resolution optimization
  - More decision variables for distributions
- Higher fidelity analysis
  - Increase blade discretization
  - Qprop
  - VABS

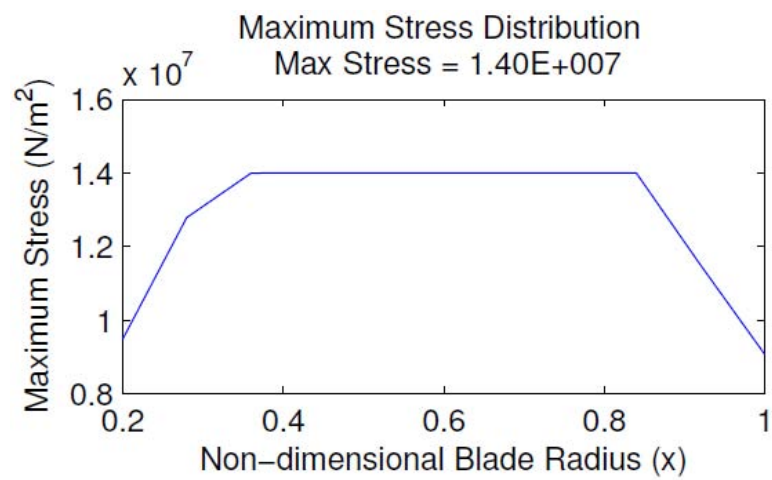
  (Qprop, VABS: Also for validation)
- Higher-powered optimization
  - DAKOTA implementation may be more powerful than the Matlab Optimization Toolbox
- Questions?

Backup slide: Design Vector (t)



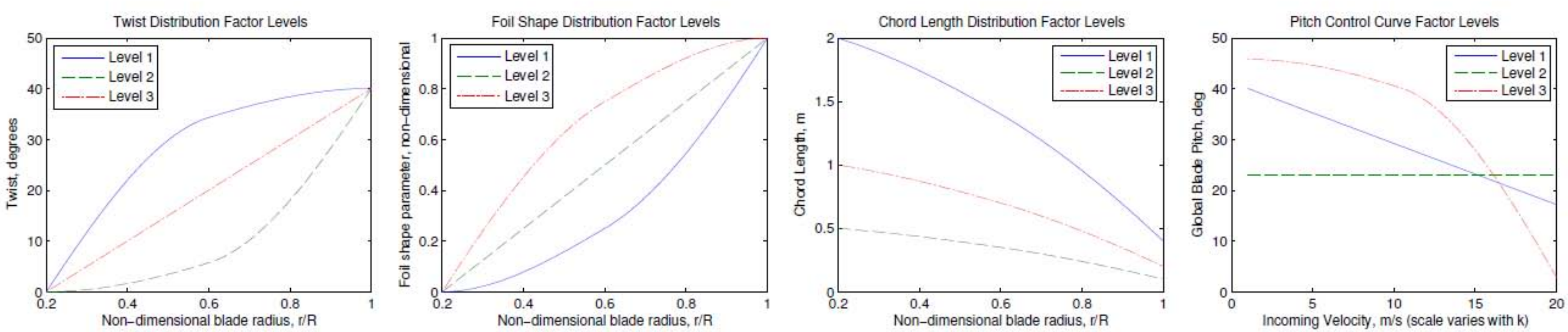


- Thickness of structural spar compensates for light structure
  - Adds leeway



Backup slide: DOE factors/levels

Factor	Variable	Levels	Level 1	Level 2	Level 3	Focus
1	R	3	16.15	14.15	12.15	Magnitude
2	$Qmax$	3	12000	16000	20000	Magnitude
3	t	3	0.004	0.01	0.02	Magnitude
4	T	3	(0.6, 0.7)	(0.1, 0.7)	(0.35, 0.7)	Shape
5	F	3	0.25	0.5	0.75	Shape
6	C	3	(2, 1.4, 0.4)	(0.5, 0.35, 1)	(1, 0.7, 0.2)	Magnitude
7	β	3	(0.7, 0.5, 0.3)	(0.4, 0.4, 0.4)	(0.8, 0.7, 0.05)	Shape
8	k	2	3	4	-	Magnitude



Backup slide: DOE Main Effects

	P_E/V_{blades}			σ^r_{max}		
Factor	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
1	-1286	1438	-152	0.112	0.983	-1.095
2	-114	865	-751	-0.962	0.685	0.277
3	3374	-783	-2591	1.895	-0.778	-1.117
4	4148	-2919	-1229	0.955	-0.183	-0.772
5	-105	8	96	0.322	0.297	-0.618
6	-670	1862	-1192	-2.410	3.592	-1.182
7	251	-2319	2069	0.087	-0.855	0.768
8	-250	250	-	-0.535	0.535	-

Best combination of factor levels (2,2,1,1,3,2,3,2)

$$P_E/V_{blades} = 20686\ \mathrm{W/m^3}$$
$$\sigma^r_{max} = 12.22\ \mathrm{MPa}$$