# HST.584 / 22.561 Problem Set #2 Solutions

Marking Scheme: Question 1 – 3 points, Question 2 – 3 points, Question 3 – 4 points
General comments: T_{1r} actually should be $T_{1\rho}$ (sorry)

1-a) Note that the decay due to $T_{1\rho}$ depends on τ, our spin
lock interval; as soon as we release the spin lock, we get
an FID as following a typical $\pi/2$ pulse. One possible way
to measure $T_{1\rho}$ is to perform a series of spin lock
experiments for various values of τ. If we measure the
maximum amplitude resulting from the FID as a function
of different τ, we can fit a curve of the form $\exp(-\tau/T_{1\rho})$
and determine $T_{1\rho}$ (the correct way to do this would be to
tack a spin echo onto the end of the spin lock so that we are not trying to measure the FID
amplitude so close to when the B_1 field is active).
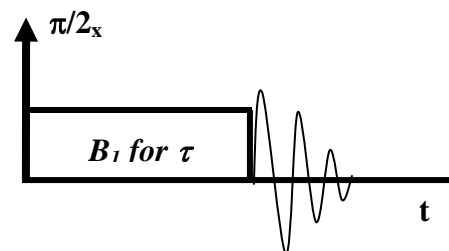
1-b) In the limit of small B_1, $T_{1\rho}$ approaches T_2^*. As B1 → 0, the effects of field
inhomogeneities become apparent and eventually dominant; this manifests as T_2^*
relaxation.

2) In class, Bruce actually drew both T_1 and T_2 as a function of τ_c, the correlation time.

Relaxation Time

$$T_1 \propto \tau_c; \omega\tau_c \gg 1$$

$$T_2 \propto \frac{1}{\tau_c}; \omega\tau_c \ll 1$$

$$T_2 \propto \frac{1}{\tau_c}; \omega\tau_c \gg 1$$

Correlation
Time
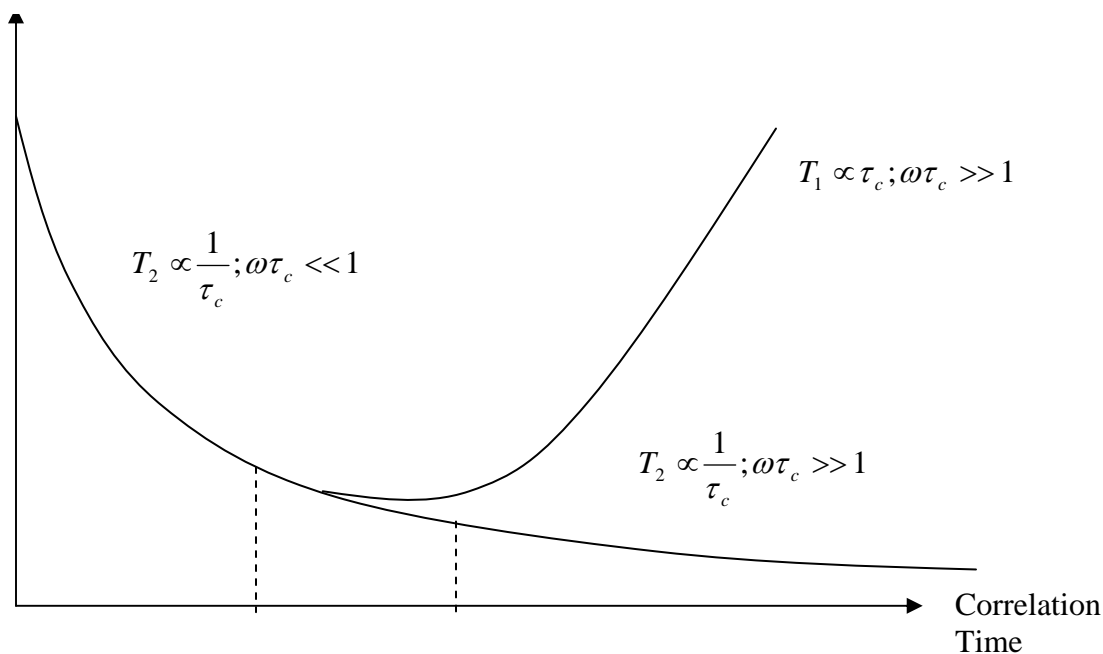
For T_2 relaxation, the J(0) term is dominant (and unique to T_2, unlike the higher ω and 2ω
contributions which appear in both T_1 and T_2). This is term responsible for dephasing,

and does not cause the spins to lose (or gain energy). Comparing liquids and solids, our spins in a liquid have a great deal more molecular motion (rotation, translation, vibration) and hence short correlation times. This case corresponds to the left side of our graph, so we see that for liquids T_1 and T_2 are of the same order of magnitude (which is exactly what we see in practice). In solids, molecular motion is much more constrained and hence the correlation time longer. This case is the right side of our figure, and hence in solids T_1 is large and T_2 very short. Another way to think of this is the idea of a random walk process, during which a spin samples the local magnetic field variations as it moves. In a liquid, the spin takes many steps and samples many different local fields (some likely to increase its phase, some likely to decrease its phase). Thus, on average the spin is more likely to remain close to its initial phase value, dephasing is thus delayed and T_2 is relatively large. In a solid, a spin does not move very much and so only samples a very few local fields that will either increase or decrease its phase. Dephasing is thus enhanced on average and T_2 is relatively short.

3-a) (*Note: I'm not going to draw the vector diagrams to show this – a real pain in Word. If you want me to show you, come talk to me in class or office hour.*) If we have perfect refocusing pulses, there is no difference between our CP and CPMG echo trains. We should detect exactly the same signal.

3-b) In the CP sequence, if we have imperfect refocusing pulses, we find that our apparent T_2 decay will be enhanced (i.e. our T_2 time constant will be shorter than its actual value). This enhancement is due to the fact that the error will accumulate with each new refocusing pulse (i.e. our net magnetization vector lies further and further away from the transverse plane). This additive effect thus decreases our signal, which manifests as enhanced T_2 decay.

3-c) The CPMG sequence differs in that refocusing pulse errors will not accumulate. If our first pulse is off by $10°$ (say $170°$ instead of the desired $180°$), our magnetization will lie off the transverse plane after the first pulse, but will be returned to the transverse plane following the second pulse. Thus our odd and even echo trains will appear to be constrained by different envelopes.

3-d) To get the most accurate measure of T_2, use a CPMG sequence and fit your T_2 decay envelope only to the even echoes.