ESD.86

Models, Data, Inference for Socio-Technical Systems

Massachusetts Institute of Technology
Engineering Systems Division

Problem Set #3

Issued: Wednesday February 28, 2007. Due: Wednesday March 7, 2007 at 10:00am.

1. Max and Min. Consider two r.v.s X and Y that are uniformly distributed and independent. Random variable X is uniform over the interval [0,1]. Y is uniform over the interval [0,2].

Let

W=Min{X, Y}

Z =Max{X, Y}

(a) Find the joint pdf for W and Z.

(b) Find the pdf for $W + Z$.

2. Random Incidence. Building a Car. Imagine that you are working on an assembly line and waiting for two components, fidgets and whoosies. As soon as you have the two components you can complete the assembly of your reconstructed classic Pierce-Arrow automobile[1]. These components arrive as independent random processes on adjacent conveyor belts. Fidgets arrive as a homogeneous Poisson process with mean inter-arrival time of 10 minutes. Whoosies arrive as a renewal process with time T_W between successive renewals found from this pdf:

$$f_{T_W}(t) = (1/2)(0.1)e^{-0.1t} + (1/2)(0.02)e^{-0.02t} \quad t \geq 0.$$

The whoosie pdf is sometimes called the hyper-geometric pdf.

(a) Find the mean time between arrivals of both fidgets and whoosies.

(b) You have been on lunch break and you arrive back at your workstation, with the partially reconstructed Pierce-Arrow, and the fidget and whoosie arrival processes in full swing.

   a. Find the mean time from your arrival back at your station until the first fidget arrives.
   b. Find the mean time from your arrival back at your station until the first whoosie arrives.
   c. Find the mean time until both have arrived and you can complete your reconstruction of the Pierce-Arrow.

---

[1] For a brief history of the Pierce-Arrow car, see http://www.antiquecar.com/index/listings/category766.htm

3. Spatial Poisson on a Line. Ambulances patrol an infinitely long straight East-West highway at spatial density of γ ambulances per mile. At any given time, the ambulances are positioned along the highway as a homogeneous spatial Poisson process. Being public safety vehicles, ambulances are allowed to make U-turns anywhere, so that their direction of travel is not relevant. You have a traffic accident at point x along the highway and with your new GPS device you summon two ambulances, the closest one to the East of your location and the closest one to the West of your location. Once, summoned, each ambulance drives from its location to your location at a constant speed of 60 miles per hour to reach your location.

(a) Determine the pdf for the time until the first ambulance arrives. Calculate the mean value.

(b) Determine the pdf for the time until the second ambulance arrives. Calculate the mean value. Compare to the mean value found in part (a). Are you surprised in any way? Comment.

4. Covered Rectangle. Homogeneous spatial Poisson processes have in two dimensions the same property that homogeneous time Poisson processes have: Given that we have conducted our probabilistic experiment and we know only the exact number of Poisson entities within a fixed area, then – with no further information – the *locations* of these entities are independent and uniformly distributed over the fixed area. This property is analogous to the unordered arrival times in a homogeneous time-Poisson process, as these unordered times are independent and uniformly distributed over the fixed time interval of interest. Suppose that the fixed area of interest is a 2-mile-by-10-mile rectangle, with sides parallel to the x and y axes, respectively. The 2-mile-by-10-mile rectangle is situated within a larger community in which a homogeneous spatial Poisson Process operates. Now suppose that the spatial Poisson process has distributed its entities over the community. We are told only that the number of Poisson entities within the rectangle is exactly 10. We define the *coverage rectangle* as the smallest rectangle within the 2-by-10 mile rectangle that contains all 10 Poisson entities and that has sides parallel to the larger rectangle. Find the expected area of the coverage rectangle.

5. Random Incidence – Cookies, etc. In your own ESD research or other professional activity, think about the random incidence problems that are explained by the "chips in chocolate chip cookies" setup. That is, in your work there is the potential for major selection bias due to this phenomenon. Identify one focused topic in your research or professional activity in which such selection bias is or could be an issue. Try to frame, formulate and solve this problem is as complete detail as you can. Use the formulas where appropriate. If you have data to support your work, even better! If you want to display results on a spreadsheet, terrific! If no such threats occur in your professional work, then think of your personal life and possibilities there for this to happen. If you still draw a blank, then do the analysis for a situation you read about in the media.