

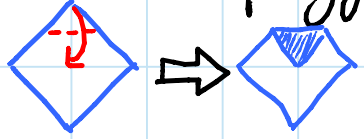
6.849

Class 2

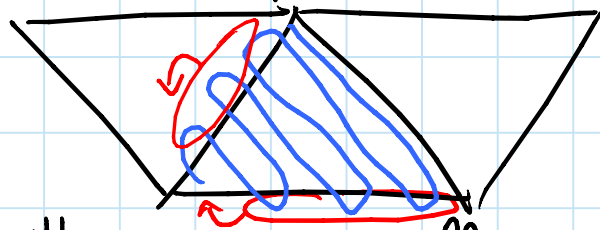
Sept. 11, 2012

- o Funny comments
- o Positive comments
 - "such a powerful theorem, so quickly"
- o Folding practice: numbers 6, 8, 4, 9!
 - cf. Jason Ku's universal alphabet & Jeannine Mosely's 4-fold alphabet
 - **PROJECT**: design 4-fold digits
- o History: why "silhouette" & "gift wrapping"?
- o Practical? strips occasional in origami
 - pretty good $n \times n$ checkerboard design (see L4)
- o Pseudopolynomial upper/lower bound? **OPEN**
 - ↳ polynomial in $n = \# \text{ vertices} + \text{edges} + \text{faces}$
 - ↳ $(nr)^c$ & geometric ratio r
 - here, $r = \frac{\text{max. diameter of face}}{\text{min. altitude of triangle}}$
 - & want to bound # folds & aspect ratio
 - upper bound claimed, but not explicit
 - $O(nr)$? $O(n+r)$?
 - presumably a lower bound e.g. $\Omega(n+r)$

o Seam placement

- convex seam patterns all possible:
 - visit seam polygons in a tour
 - transition increases/decreases width of strip via lg ratio width gadgets & offsets strip to "cover" next polygon
- some nonconvex possible: 
- OPEN: which?

o Hide gadget via simple folds? (some-layers)



mountains
might collide

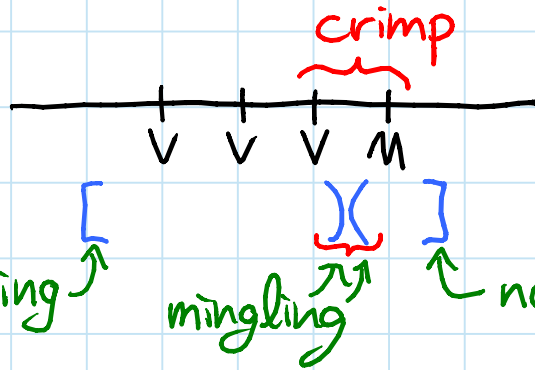
- silhouette easy: valley fold, not mountain
- OPEN: 2-color pattern?
 - idea: bicolor turn gadget/excess
 - TRY TO SOLVE
- OPEN: convex seam placement

Simple Folds: [Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000/2004]

o Motivation: bending rigid material

o Definition: single line segment
 $\pm 180^\circ$ rotation
no collision during motion

o Example:

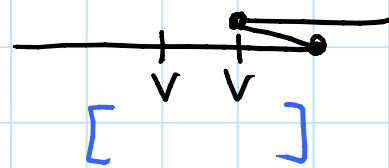


is mingling
(but not "forever")

not left mingling

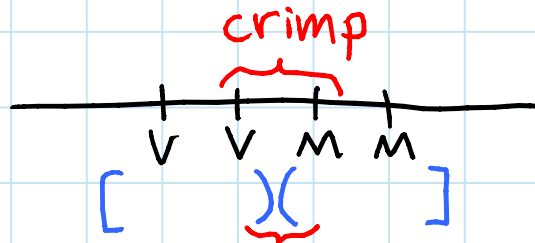
mingling

not right mingling

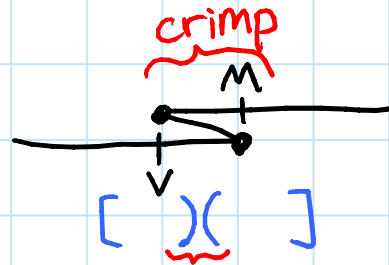


is not mingling
& not flat foldable

Another:



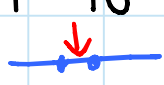
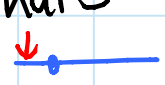


is mingling
& flat foldable



is mingling
& flat foldable



done

- o Algorithm: (NEW) (covered in C3)
- - search (left to right) for segment that's crimpable  OR end foldable 
 - if none found: STOP ~ not flat foldable
 - else: do fold  → 
- merge segments $x, y, z \rightarrow x-y+z$
 go back one segment (left of x)
 continue search

Correctness:

- doing fold changes foldability only of adjacent segments
- ⇒ enough to back up 1 step

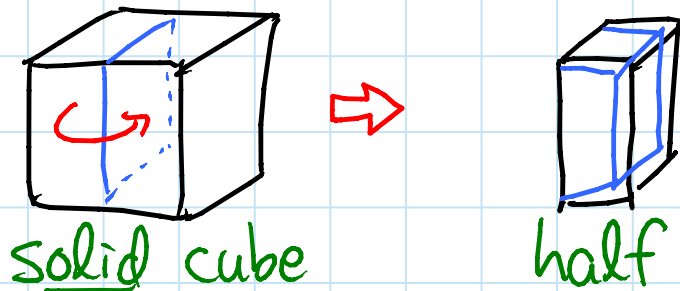
Running time: $O(n)$

- ↑ # creases
- # right steps = $n + \underbrace{\# \text{ left steps}}_{= \# \text{ folds done}}$
 - $\leq 2n$

Amortization: charge left steps to fold just done)

- o Every mountain-valley pattern can be made flat foldable by adding creases:
 - between consecutive MM's add V
 - & between consecutive VV's add M
 - ⇒ alternating M/V
 - ⇒ flat foldable
 - (globally smallest segment is crimpable)

- o d-dimensional paper
 - ⇒ (d-1)-dimensional creases
 - & (d+1)-dimensional ambient space
 - d-dimensional = flat folding



MIT OpenCourseWare
<http://ocw.mit.edu>

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
Fall 2012

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