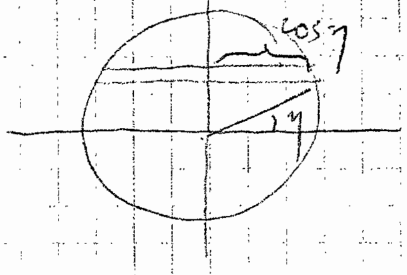
Circle:



$$\left.\begin{aligned} 2\pi r\,\delta S &= \delta O \\ 2\pi \cos\eta\,\delta\eta &= \delta S \end{aligned}\right\} \quad \frac{\delta S}{\delta O} = \frac{\cos\eta}{r}\cdot\frac{\delta\eta}{\delta S} = \frac{\cos\eta}{r} K_G$$

$$\boxed{K = \frac{\cos\eta}{r} K_G} \qquad K_G = \frac{d\eta}{ds}$$

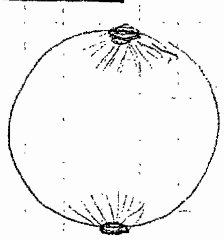
$$r = r(\eta) : \quad \frac{dr}{ds} = -\sin\eta \qquad \cos\eta \frac{d\eta}{ds} = \frac{d^2 r}{ds^2} \qquad K = -\frac{r_{ss}}{r}$$

$$\frac{dr}{dz} = -\tan\eta \qquad \sec^2\eta \frac{d\eta}{ds} = -\frac{d^2 r}{dz^2}\frac{dz}{ds}$$

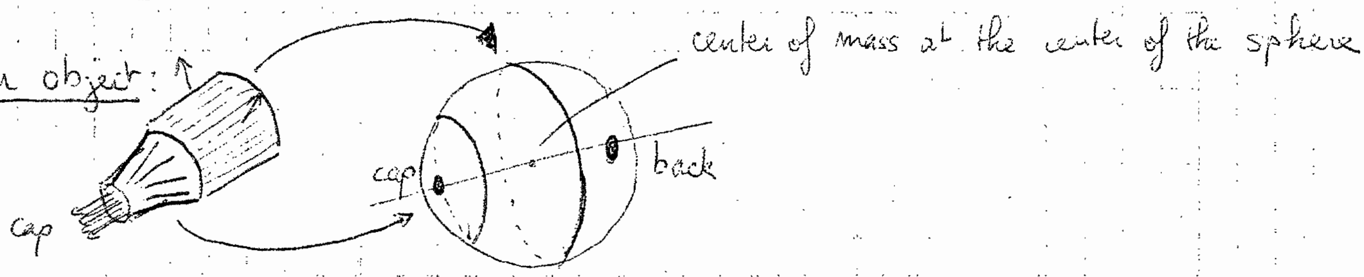
$$\frac{dz}{ds} = \cos\eta \quad : \quad K_G \cos\eta = \frac{d\eta}{ds}\cos\eta = -r_{zz}\cos^4\eta$$

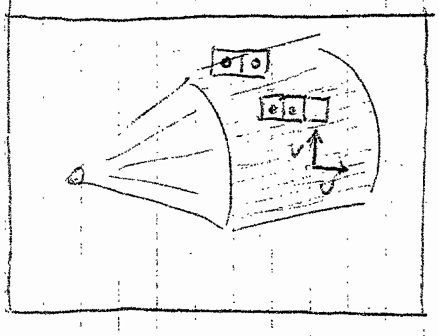
$$K = -\frac{r_{zz}}{r(1+r_z^2)^2}$$

Back to torus: $G(\xi,\eta) = 2\rho R \sec\eta$



Another object:





$\underline{r}(u,v)$

$\underline{r}_u \times \underline{r}_v \rightarrow$ direction of normal $\hat{\underline{n}}$

$\| \underline{r}_u \times \underline{r}_v \| =$ area of the facet (value on the sphere)

Tessellation of the sphere

1. cells have same area
2. cells have same shape
3. regular pattern
4. rounded shapes
5. some rotations bring self alignment

So let us look for candidates...

Platonic solids

- tetra – 4
- hexa – 6
- octa – 8
- dodeca – 12
- icosa – 20

not very good...

→ Archimedean Solids

truncated icosahedron

Pentakis dodecahedron

$5 \times 12 = 60$ facets

dual of truncated icosahedron

Duality

Vertex $\longleftrightarrow$ Face
Edge $\longleftrightarrow$ Edge ($\perp$)
Face $\longleftrightarrow$ Vertex

tetrahedron $\circlearrowleft$
cube $\longleftrightarrow$ octahedron
dodecahedron $\longleftrightarrow$ icosahedron

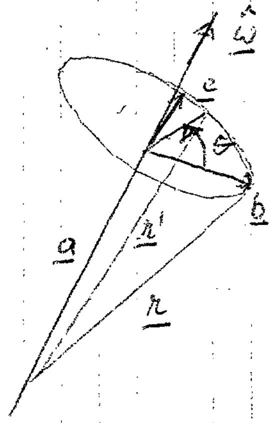
Rotation Representations

Euler Angles	3
Axis & Angle	2+1
Rot. matrix	9
Gibbs Vector	3
Quaternions	4
Euler/Rodrigues	4
Cayley-Klein	4
Pauli-spin mat.	2 complex

<u>Desired:</u>

no singularities
unredundant
easy vector rotation
easy composition
"space of rotations"
tessellation
Calculus applied to stat.

Rodrigue's formula



$$\underline{r} = \underline{a} + \underline{b}$$
$$\underline{r}' = \underline{a} + \underline{b}\cos\theta + \underline{c}\sin\theta$$
$$\underline{a} = (\underline{r}\cdot\hat{\underline{\omega}})\hat{\underline{\omega}}$$
$$\underline{b} = \underline{r} - (\underline{r}\cdot\hat{\underline{\omega}})\hat{\underline{\omega}} = (\hat{\underline{\omega}}\cdot\hat{\underline{\omega}})\underline{r} - (\underline{r}\cdot\hat{\underline{\omega}})\hat{\underline{\omega}}$$
$$= \hat{\omega} \times (\underline{r} \times \hat{\underline{\omega}})$$
$$\underline{c} = \hat{\underline{\omega}} \times \underline{b}$$

$$\boxed{\underline{r}' = (1-\cos\theta)(\underline{r}\cdot\hat{\underline{\omega}})\hat{\underline{\omega}} + \cos\theta\, \underline{r} + \sin\theta\,(\hat{\underline{\omega}} \times \underline{r})}$$

$$\underline{r}' = \underbrace{\left((1-\cos\theta)\,\omega\omega^T + \cos\theta\, I + \sin\theta\, \Omega\right)}_{R} \underline{r} \qquad \Omega = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$$

$$r_{11} = (1-\cos\theta)\,\omega_x^2 + \cos\theta + 0$$
$$r_{12} = (1-\cos\theta)\,\omega_x\omega_y + 0 + (-\sin\theta\,\omega_z)$$
$$\vdots$$

$\Longrightarrow$ We've build a bridge between "Matrix" and "Axis & Angle" notations.