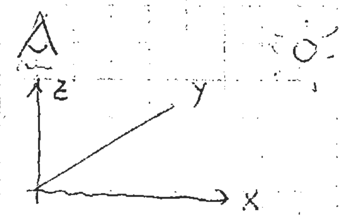
Example Hapke's $\sqrt{\dfrac{\cos\theta_i}{\cos\theta_e}}$ $R(p,q)$?



$\boxed{6.866 \quad 09\backslash 28}$

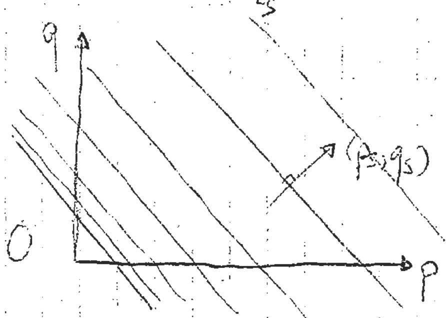
$\cos\theta_i = \hat{n}\cdot\hat{s}$ $\cos\theta_e = \hat{n}\cdot\hat{v}$

with $\hat{v} = \hat{z}$

and $\hat{n} = \dfrac{(-p, -q, 1)^T}{\sqrt{1+p^2+q^2}}$ thus $\hat{n}\cdot\hat{v} = \dfrac{1}{\sqrt{1+p^2+q^2}}$

$$R(p,q) = \sqrt{\frac{1+p_s p + q_s q}{\underbrace{\sqrt{1+p_s^2+q_s^2}}_{r_s}}}$$

$R = C^{te} \Rightarrow R^2 r_s = 1 + p_s p + q_s q \rightarrow$ line



Get the shape of the surface ?

We only have the surface reflectance map.
2 unknowns, 1 measurement
(p,q)

Additional constraint?

$$E_1(p,q) = R_1(p,q) \quad (\text{image } 1)$$

$$E_2(p,q) = R_2(p,q) \quad (\text{image } 2)$$

→ take two pictures with two different conditions.

↳ (p_{s_1}, q_{s_1}) (p_{s_2}, q_{s_2})

$$E_1 = \frac{1}{\sqrt{c_1}} \sqrt{1 + p_{s_1} p + q_{s_1} q} \qquad E_2 = \dots$$

$$\begin{cases} p_{s_1} p + q_{s_1} q = c_1 E_1^2 - 1 \\ p_{s_2} p + q_{s_2} q = c_2 E_2^2 - 1 \end{cases}$$

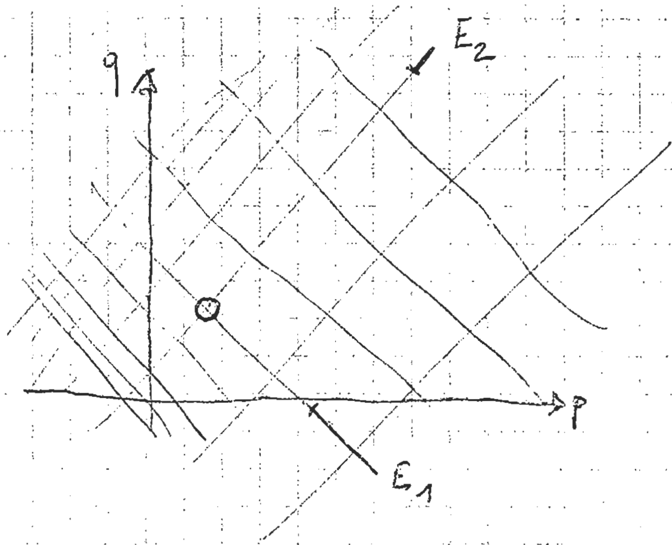
or:

$$\begin{vmatrix} p_{s_1} & q_{s_1} \\ p_{s_2} & q_{s_2} \end{vmatrix} \begin{vmatrix} p \\ q \end{vmatrix} = \begin{vmatrix} c_1 E_1^2 - 1 \\ c_2 E_2^2 - 1 \end{vmatrix}$$

$$\det M = p_{s_1} q_{s_2} - p_{s_2} q_{s_1} \neq 0 \qquad \boxed{\frac{p_{s_1}}{q_{s_1}} \neq \frac{p_{s_2}}{q_{s_2}}}$$

↳ have to <u>really</u> change direction!

problem for the moon!



If it is almost parallel, the error is high.

Photommetric stereo (P. S.) Lambertian

stereo = solid

$$E_1 = \frac{1 + p_{s_1} p + q_{s_1} q}{\sqrt{1 + p_{s_1}^2 + q_{s_1}^2}\sqrt{1+p^2+q^2}} \qquad E_2 = \frac{1 + p_{s_2} p + q_{s_2} q}{\sqrt{1 + p_{s_2} p + q_{s_2} q}\sqrt{1+p^2+q^2}}$$

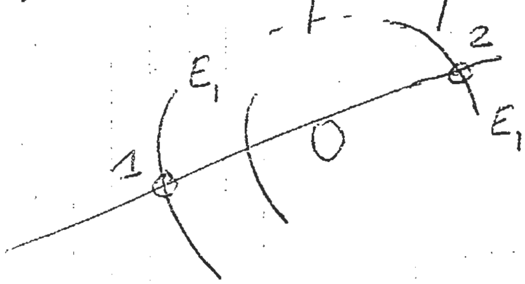
Bezout's theorem

Maximum number of solutions is the product of orders

$2 \times 2 = 4$ solutions

$$\frac{E_1}{E_2} = \underbrace{\sqrt{\frac{1+p_{s_2}^2+q_{s_2}^2}{1+p_{s_1}^2+q_{s_1}^2}}}_{r_2/r_1} \cdot \frac{1+p_{s_1}p+q_{s_1}q}{1+p_{s_2}p+q_{s_2}q}$$

$\hookrightarrow$ linear in p and q



It's simpler and more accurate to use a 3rd measurement.

Albedo ρ "non-ideal" surface reflectance

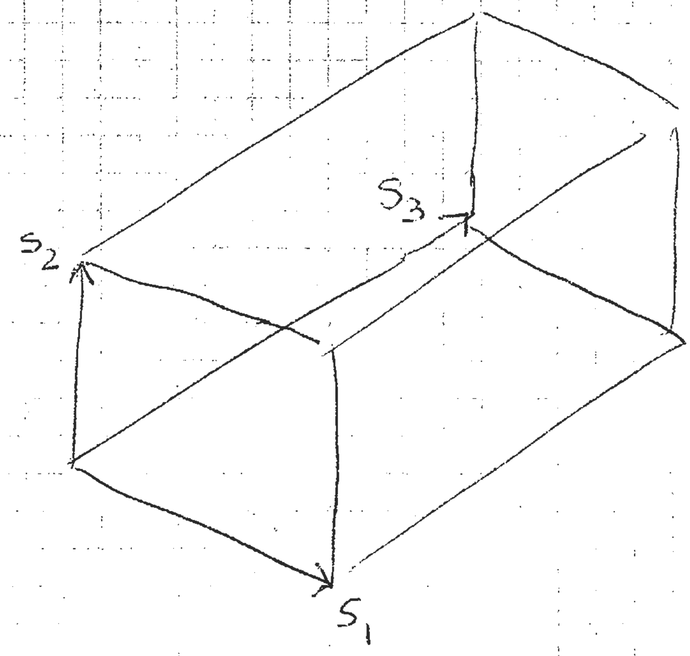
example for a Lambertian:

$$\left.\begin{aligned} E_1 &= \underline{n}\cdot\hat{s}_1 \\ E_2 &= \underline{n}\cdot\hat{s}_2 \\ E_3 &= \underline{n}\cdot\hat{s}_3 \end{aligned}\right\} \qquad \begin{aligned} \underline{n} &= \rho\,\hat{\underline{n}} \\ \underline{s_i} &= S_i\,\hat{\underline{s_i}} \end{aligned}$$

thus: $E_1 = \rho S_1 \underbrace{(\hat{n}\cdot\hat{s}_1)}_{\cos\theta_1}$

3 unknowns: ρ, p, q ⊕ 3 equations = OK!

$$\underbrace{\begin{vmatrix} \underline{s_1}^T \\ \underline{s_2}^T \\ \underline{s_3}^T \end{vmatrix}}_{S} \underline{n} = \underbrace{\begin{vmatrix} E_1 \\ E_2 \\ E_3 \end{vmatrix}}_{\underline{E}}$$



$$\underline{n} = S^{-1} E$$

$$\hat{\underline{n}} = \underline{n}/\rho \qquad (\rho = \|\underline{n}\|)$$

$$\det(S) = \underline{s_1} \cdot (s_2 \wedge s_3) = [\, \underline{s_1} \;\; \underline{s_2} \;\; \underline{s_3} \,]$$

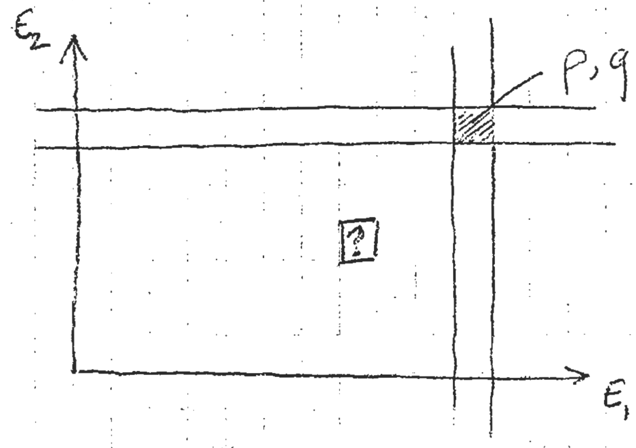
$[\underline{s_1} \; \underline{s_2} \; \underline{s_3}] = 0 \iff (\underline{s_1}, \underline{s_2}, \underline{s_3})$ on the same plane.

$V = \det S$ is maximum when $s_1 \perp s_2 \perp s_3$.

On the other hand, we want to minimize the self-shadow
$\rightarrow$ tradeoff

Practice / Numerical:

LUT



How do we construct the LUT?

→ use a patch and index the table

→ use a calibration object

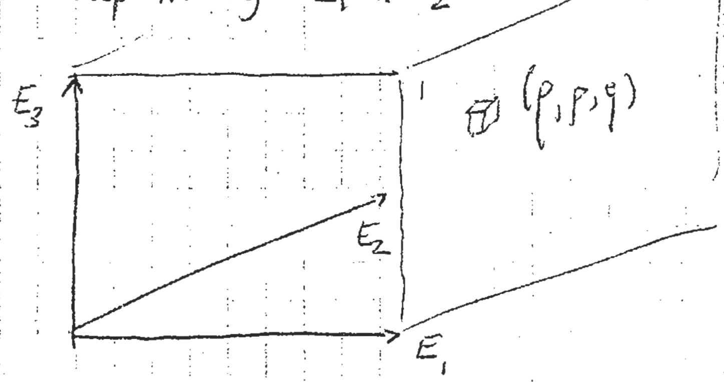
Size : 256 × 256   truncable : 64 × 64

$\boxed{?}$ → some unknown

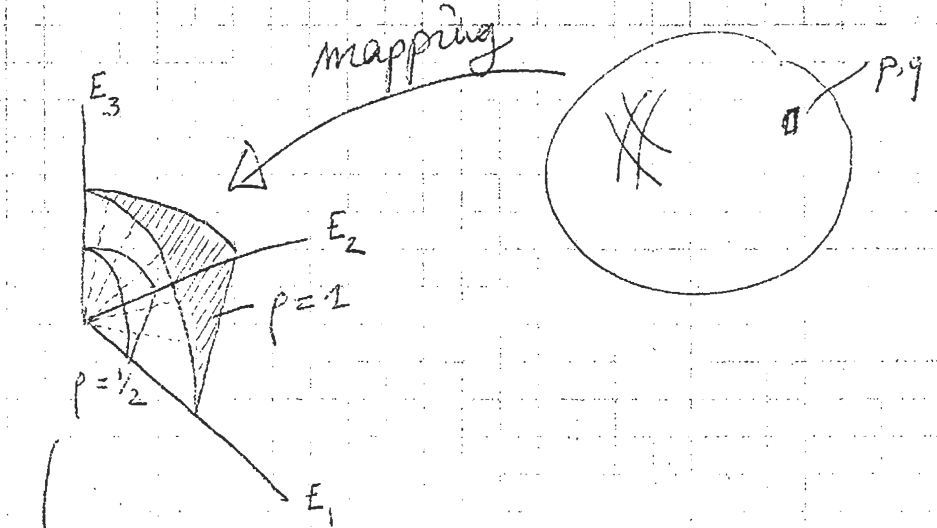
Sol 1: interpolate

Sol 2: step through E_1 & E_2

3D LUT:



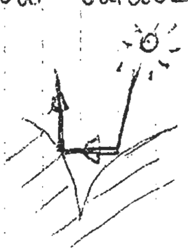
$$E_i(x, y) = R_i(p, q)$$



we can fill in the 3D LUT!

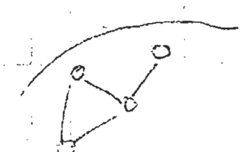
$\boxed{6.866 \quad 09/30}$

Now, if the surface has a constant albedo, 2 meas. are enough but 3 meas. gives error info. (onion slice in E^3 space) thus we can sort out outliers (shade, bogues, etc.) as well as mutual illumination.

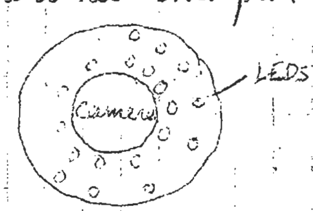


2 images, what about more?

ex @ CMU: half a sphere with light sources



another example: a circle of LEDs:



and you get a better fit.