Problem Set #7

Issued: Monday, April 7, 2003
Due: Monday, April 14, 2003

Problem 1

Resistivity is found to vary in a systematic fashion across a 200 mm wafer. Our goal here is to study the effect of different sampling patterns for the modeling of resistivity across the wafer, and for determining a single "nonuniformity" metric.

For comparison and study, we will create a synthetic dataset. Suppose that resistivity ρ varies as:

$$\rho = 50 - \frac{(x-20)^2}{50^2} - \frac{(y-15)^2}{70^2} + N(0, 0.49)$$

where (0,0) is the center of the wafer, and coordinates are in mm.

(a) Consider first a sample of 25 points on the wafer, consisting of an outer ring of 8 points (all 8 points distributed evenly) at a fixed radius of 75 mm, a middle ring of 8 points at a radius of 50 mm, an inner ring of 8 points at a radius of 25 mm, and a center point. Create a table with x, y, and ρ values for your sample. Plot the location of the datapoints, and plot contours of resistivity for your sample (most packages are able to generate such contour plots).

(b) Next produce a regular rectangular sampling of your wafer consisting of a 5x5 grid of evenly spaced points (e.g. at 0, ± 40, and ± 80 mm). Again, plot the locations and contours of resistivity for your 25 sample points, and create a table with x, y, and ρ values for your sample. If you can exclude those points falling outside the boundary of the wafer edge (i.e. those points with radius greater than 100mm), you should do so. For example, JMP allows one to "exclude" such points.

(c) For each sampling plan, fit a regression model of the form:

$$\rho = a + bx + cy + dxy + ex^2 + fy^2$$

Compare each regression to the known functional form, and comment. Which model parameters are significant? Do these match the known systematic variation function? Are the confidences in these parameters similar in the radial and rectangular patterns?

(d) A common uniformity metric U is the "signal to noise ratio". Often this is inverted to express the *nonuniformity* NU as $\mathrm{NU} = \sigma/\mu$ where μ is estimated as the sample mean and σ is estimated as the sample standard deviation. Calculate NU for each of your two sampling plans, where equal weighting is used for each of the sample points (that is, no compensation for "area" representation is attempted), and comment.

(e) As an estimate of the “true” nonuniformity, calculate NU for a 29x29 sample across the wafer (where the 29 points fit across the full 200 mm wafer diameter), being careful to discard points outside the boundaries of the wafer. Fit a spatial regression model using this dense sampling, and compare to that generated using the radial and rectangular 25 point samples above. Taking this 29x29 sample as an approximation to the “true” nonuniformity, comment on how well the 25 point patterns perform in estimating nonuniformity.

(f) Extra Credit. One approach to improve the radial pattern is to weight each sample point with the “area” which that point represents on the wafer. Determine appropriate weights, and perform a weighted linear regression for the 25 point radial pattern. Does this result in a better estimate of NU? Another approximation approach is to change the number of points used at each radius. Describe how you might select the number of points at different radii in order to best estimate wafer level NU.