Supplement 1 - <u>Semiconductor Physics Review</u> - Outline

- ## The Fermi function and the Fermi level
    ### The occupancy of semiconductor energy levels

- ## Effective density of states
    ### Conduction and valence band density of states
    #### 1. General
    #### 2. Parabolic bands

- ## Quasi Fermi levels
    ### The concept and definition

    ### Examples of application
    #### 1. Uniform electric field on uniform sample
    #### 2. Forward biased p-n junction
    #### 3. Graded composition p-type heterostructure
    #### 4. Band edge gradients as effective forces for carrier drift

Refs: R. F. Pierret, <u>Semiconductor Fundamentals 2nd. Ed.</u>, (Vol. 1 of the Modular Series on Solid State Devices, Addison-Wesley, 1988); TK7871.85.P485; ISBN 0-201-12295-2.
          S. M. Sze, <u>Physics of Semiconductor Devices</u>   (see course bibliography)
          Appendix C in Fonstad   (handed out earlier; on course web site)

Fermi level and quasi-Fermi Levels - review of key points

Fermi level: In thermal equilibrium the probability of finding an energy level at E occupied is given by the Fermi function, f(E):

$$f(E) = 1 \Big/ \left(1 + e^{[E - E_f]/kT}\right)$$

where E_f is the Fermi energy, or level. In thermal equilibrium E_f is constant and not a function of position.

The Fermi function has the following useful properties:

$$f(E) \approx e^{-[E - E_f]/kT} \qquad for\ (E - E_f) >> kT$$

$$f(E) \approx 1 - e^{[E - E_f]/kT} \quad for\ (E - E_f) << -kT$$

$$f(E_f) = 1/2 \qquad for\ E = E_f$$

These relationships tell us that the population of electrons decreases exponentially with energy at energies much more than kT above the Fermi level, and similarly that the population of holes (empty electron states) decreases exponentially with energy when more than kT below the Fermi level.

A final set of useful Fermi function facts are the values of f(E) in the limit of T = 0 K:

$$\lim_{T=0} f(E) = 1 \qquad for\ E < E_f$$

$$f(E) = 1/2 \qquad at\ E = E_f$$

$$\lim_{T=0} f(E) = 0 \qquad for\ E > E_f$$

Effective densities of states: we can define an effective density of states for the conduction band, N_c, as

$$N_c \equiv \int_{E_c}^{\infty} \rho(E) e^{-[E-E_c]/kT} dE$$

and an effective density of states for the valence band, N_v, as

$$N_v \equiv \int_{-\infty}^{E_v} \rho(E) e^{[E-E_v]/kT} dE$$

where **ρ**(E) is the electron density of states in the semiconductor.

If the energy bands are parabolic, i.e., when the density of states depends quadraticly on the energy away from the band edge, we find simple relationships between the densities of states and the effective masses:

$$\text{If } \rho(E) = \sqrt{2(m_e^*)^3(E - E_c)}\Big/\pi^2 h^3 \quad \text{when } E > E_c,$$

$$\text{then } N_c = 2\left[2\pi m_e^* kT/h^2\right]^{3/2}$$

$$\text{and if } \rho(E) = \sqrt{2(m_h^*)^3(E_v - E)}\Big/\pi^2 h^3 \quad \text{when } E < E_v,$$

$$\text{then } N_v = 2\left[2\pi m_h^* kT/h^2\right]^{3/2}$$

When $(E_c$-$E_f)$>>kT, we can write the thermal equilibrium electron concentration in terms of effective density of states of the conduction band and the separation between the Fermi level and the conduction band edge, E_c, as:

$$n_o(x) = N_c(x)\, e^{-[E_c(x) - E_f]/kT}$$

Similarly when $(E_f$-$E_v)$>>kT we can write:

$$p_o(x) = N_v(x)\, e^{-[E_f - E_v(x)]/kT}$$

Note: In homogeneous material N_c, and N_v do not depend on x.

Quasi-Fermi levels: When a semiconductor is not in thermal equilibrium, it is still very likely that the electron population is at equilibrium within the conduction band energy levels, and the hole population is at equilibrium with the energy levels in the valence band. That is to say, the population on electrons is distributed in the conduction band states with the Boltzman factor: $e^{-[E-E_{fn}]/kT}$

Here E_{fn} is the effective, or quasi-, Fermi level for electrons. Similarly, there is a quasi-Fermi level for holes, E_{fp}, and the holes are distributed in the valence band states as: $e^{-[E_{fp}-E]/kT}$

The quasi-Fermi levels for electrons and holes, E_{fn} and E_{fp}, are not in general equal. To find them we usually begin with n(x) and p(x), and write them in terms of the conduction and valence band densities of states and the quasi-Fermi levels:

$$For\ example, we\ write\ n(x) = N_c(x)e^{-[E_c(x)-E_{fn}(x)]/kT}$$

$$This\ then\ defines\ E_{fn}:\ E_{fn}(x) \equiv E_c(x) - kT\ln[N_c(x)/n(x)]$$

$$We\ define\ E_{fp}\ similarly:\ E_{fp}(x) \equiv E_v(x) + kT\ln[N_v(x)/p(x)]$$

Quasi-Fermi levels, cont.:

A very important finding involving quasi-Fermi levels is that we can write the electron and hole currents in terms of the gradients of the respective quasi-Fermi levels, at least in the low field limit where drift mobility is a valid concept. We find:

$$J_n(x) = \mu_e \, n(x) \, \partial E_{fn}(x)/\partial x$$

and

$$J_p(x) = \mu_h \, p(x) \, \partial E_{fp}(x)/\partial x$$

Examples:

A. Uniformly doped n-type semiconductor with uniform E-field

At low to moderate E-fields, the populations are not disturbed from their equilibrium values, and we have

$$n(x) \approx n_o \approx N_d \quad and \quad p(x) \approx p_o = n_i^2/N_d$$

$$Also, \; E_{fn} \approx E_{fp} \approx E_f - q\phi(x), \quad so:$$

$$J_e \approx \mu_e n_o(-q\,\partial\phi/\partial x) = q\mu_e n_o F_x \quad and \quad J_e \approx q\mu_h p_o F_x$$

As expected, the currents are the respective drift currents.

B. P-side of forward biased n⁺-p junction, long-base limit:

Diode diffusion theory gives us n(x) on the p-side[*]:

$$n(x) = n_{op}[(e^{qv_{ab}/kT} - 1)e^{-x/L_e} + 1], \; where \; n_{op} = n_i^2/N_{Ap}$$

When $v_{AB} \gg kT$, and x is not many L_e, we can approximate n(x) as :

$$n(x) = n_{op}[(e^{qv_{ab}/kT} - 1)e^{-x/L_e} + 1] \approx n_{op}e^{qv_{ab}/kT}e^{-x/L_e}$$

from which we find:

$$E_{fn}(x) = E_c + kT\ln[n(x)/N_c]$$

$$\approx E_c + kT\ln[n_o/N_c] + qv_{ab} - kT \; x/L_e$$

$$\approx E_{fo} + qv_{ab} - kT \; x/L_e$$

We see that $E_{fn}(x)$ is qv_{AB} higher than the equilibrium Fermi level, E_{fo}, at the edge of the depletion region on the p-side, and decreases linearly going away from the junction. Farther away from the junction, where x is many L_e, n(x) approaches n_{op}, and $E_{fn}(x)$ approaches E_{fo}.

Finally, notice that for low-level injection, $p(x) \approx p_{po}$, and $E_{fp} \approx E_{fo}$.

Quasi-Fermi levels - Illustrating examples A and B

Figure C-8 from Fonstad, Microelectronic Devices and Circuits with quasi-Fermi levels added:

Example A:

$$E_{fn} \approx E_{fp} \approx E_{fo}$$

(a)

Example B:

(b)
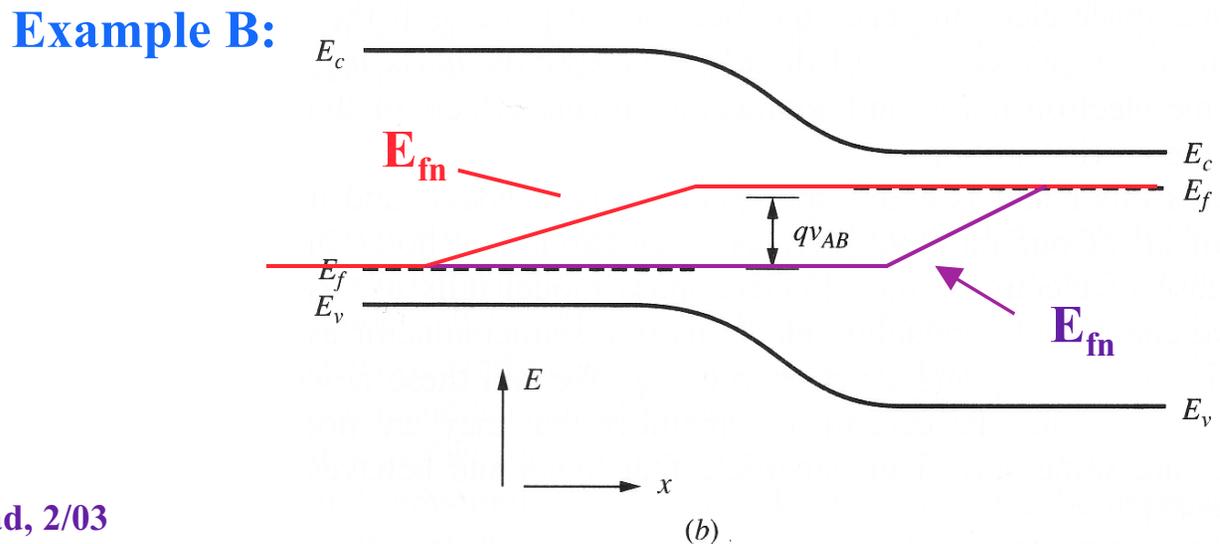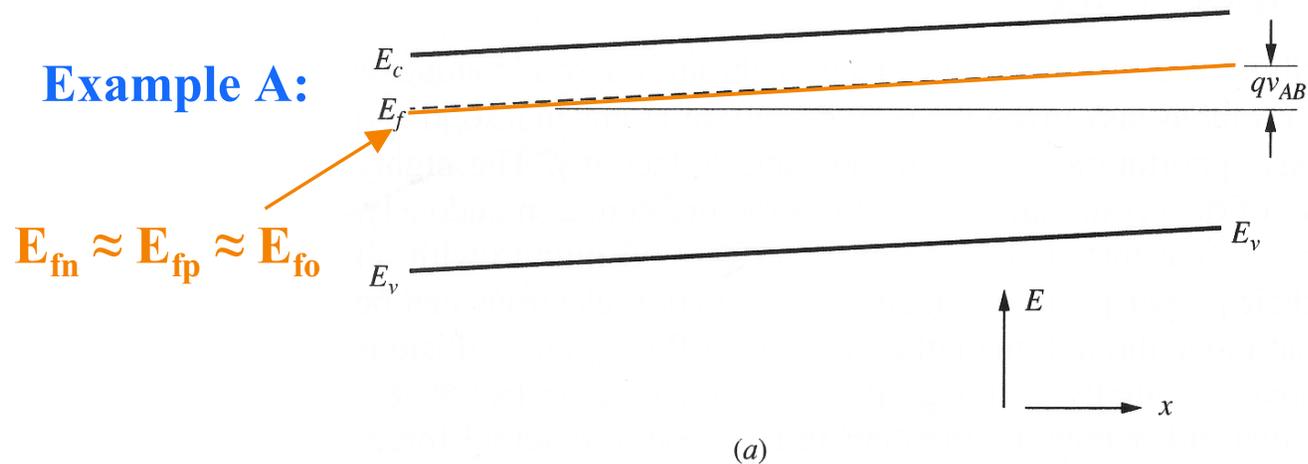
C. Graded composition p-type heterostructure with uniform low level electron injection.

Assume the grading is from E_{g1}, X_1 @ x = 0, and E_{g1}, X_1 @ x = L.

$$E_g(x) = E_{g1} + x(E_{g2} - E_{g1})/L; \qquad \chi(x) = \chi_1 + x(\chi_2 - \chi_1)/L$$

In thermal equilibrium the Fermi level, E_{fo}, is flat, and the valence band edge is flat:

$$E_v = E_{fo} - kT \ln(N_v / N_{Ap})$$

whereas the conduction band edge slopes: $\quad E_c(x) = E_v - E_g(x)$

With low-level electron injection, $n(x) \approx n'$ ($\gg n_{po}$):

Hole population is changed insignificantly, and $E_{fp}(x) \approx E_{fo}$
Electron population is now n', and so

$$E_{fn}(x) = E_c + kT\ln[n'/N_c] \approx E_v + E_g(x) + kT\ln[n'/N_c]$$
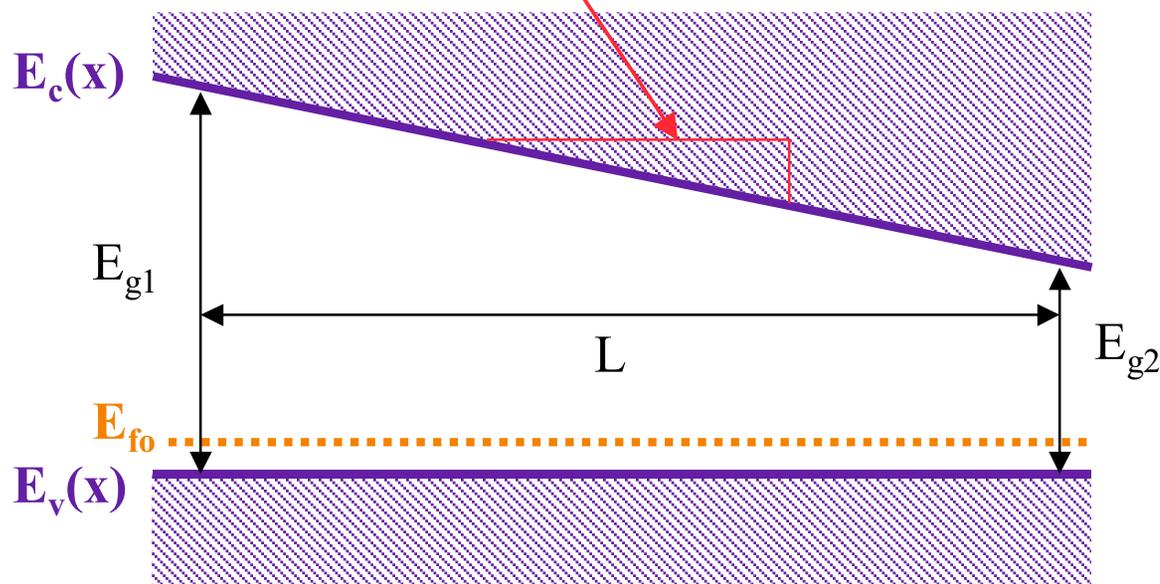
Using this to get $J_e(x)$, we find

$$J_e(x) \approx \mu_e\, n'\, \partial E_g(x)\big/\partial x = q\mu_e\, n'\, F_{e,eff}, \; where\; F_{e,eff} \equiv q^{-1}\partial E_g(x)\big/\partial x$$

From this we see that the band gap grading acts like an effective electric field acting on the electrons (but not on the holes)!

Quasi-Fermi levels - Illustrating example C

Example C:

$$F_{e,eff} \equiv q^{-1}\partial E_c(x)/\partial x = -(E_{g1} - E_{g2})/qL$$

$E_c(x)$

E_{g1}

E_{g2}

L

E_{fo}

$E_v(x)$

D. General meaning of band-edge gradings

In general we can write the electron quasi-Fermi level as:

$$E_{fn}(x) = E_c(x) + kT \ln[n(x)/N_c(x)]$$

and thus in general we can write the electron current as:

$$J_e(x) = \mu_e \, n(x) \, \partial E_{fn}(x)\big/\partial x$$

$$= \mu_e \, n(x) \, \frac{\partial E_c(x)}{\partial x} + \mu_e \, kT \, \frac{\partial n(x)}{\partial x} + \mu_e \, kT \, \frac{n(x)}{N_c(x)} \, \frac{\partial N_c(x)}{\partial x}$$

typically small

$$= \sigma_e \, \frac{q^{-1}\partial E_c(x)}{\partial x} + D_e \left(\frac{\partial n(x)}{\partial x} + \frac{n(x)}{N_c(x)} \, \frac{\partial N_c(x)}{\partial x} \right)$$

Drift Term   Diffusion Term

[Note: In getting this we have used the Einstien relation and definition of conductivity:

$$\mu_e \, kT = q \, D_e, \quad and \quad \sigma_e = q \, \mu_e \, n(x)$$

]

From our final result we see that the gradient in the conduction band edge is the force leading to electron <u>drift</u>, while the gradient in the carrier and density of states concentrations are the <u>diffusion</u> force.

D. cont.

   We obtain the corresponding result for holes if we similarly substitute valence band quantities for conduction band quantities. Begin with:

$$E_{fp}(x) = E_v(x) + kT \ln[p(x)/N_v(x)]$$

and thus

$$J_h(x) = \mu_h \, p(x) \, \partial E_{fp}(x)/\partial x$$

$$= \mu_h \, p(x) \, \frac{\partial E_v(x)}{\partial x} + \mu_h \, kT \, \frac{\partial p(x)}{\partial x} + \mu_h \, kT \, \frac{p(x)}{N_v(x)} \, \frac{\partial N_v(x)}{\partial x}$$

typically small

$$= \sigma_h \, \frac{q^{-1}\partial E_v(x)}{\partial x} + D_h \left(\frac{\partial p(x)}{\partial x} + \frac{p(x)}{N_v(x)} \, \frac{\partial N_v(x)}{\partial x} \right)$$

Drift Term   Diffusion Term

   Now we see that the gradient in the valence band edge is the force leading to hole **drift**, while the gradient in the carrier and density of states concentrations are the **diffusion** force.

   Summarizing, the conduction and valence band-edge gradients can be viewed as effective electric fields for electrons and holes, respectively:

$$F_{e,eff} = q^{-1}\partial E_c(x)/\partial x \quad and \quad F_{h,eff} = q^{-1}\partial E_v(x)/\partial x$$