

Lecture 14 - p-n Junction (*cont.*)

March 8, 2007

Contents:

1. Ideal p-n junction out of equilibrium (*cont.*)

Reading assignment:

del Alamo, Ch. 7, §7.2 (§7.2.3)

Announcements:

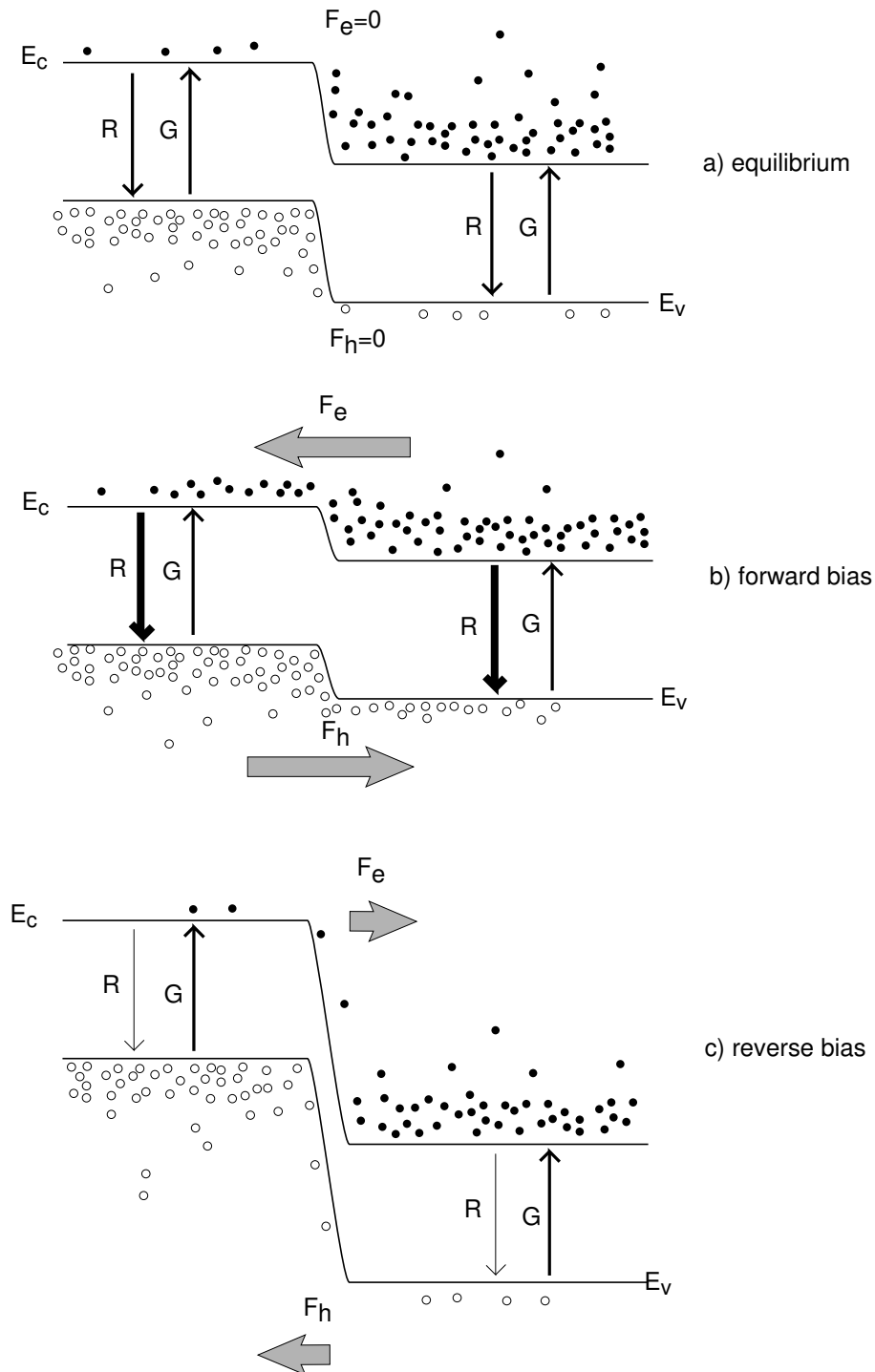
Quiz 1: **March 13**, 7:30-9:30 PM; lectures #1-12
(up to SCR-type transport). Open book.
Calculator required.

Key questions

- What are the dominant physics of current flow in a p-n junction under bias?
- What underlies the rectifying behavior of the p-n junction?
- What are the key assumptions that allow the development of a simple model for the I-V characteristics of a p-n diode?
- What are the key dependencies of the current-voltage characteristics of the p-n diode?

1. Ideal p-n junction out of equilibrium (*cont.*)

□ I-V characteristics



★ In TE:

- balance between electron and hole flows across SCR
- balance between G and R in QNR's
- $I = 0$

★ In forward bias:

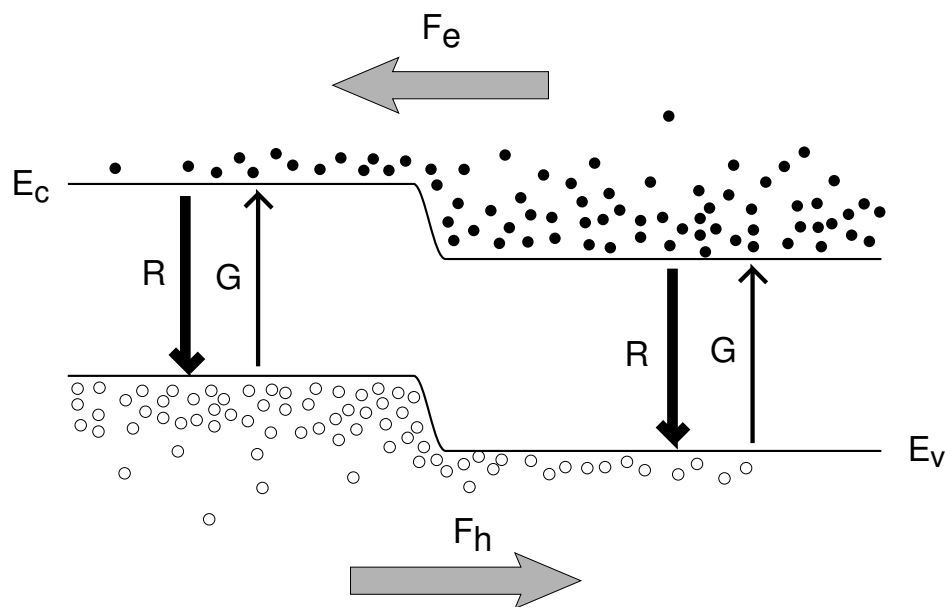
- energy barrier to minority carriers reduced
- *minority carrier injection*
- $R > G$ in QNR's
- $I \sim e^{qV/kT}$

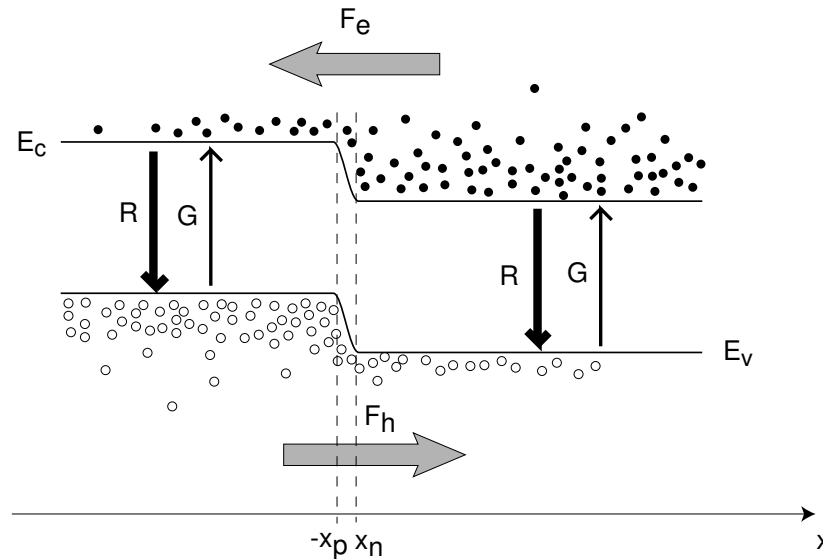
★ In reverse bias:

- energy barrier to minority carriers increased
- *minority carrier extraction*
- $G > R$ in QNR's
- I saturates to a small value

Key thinking to construct model for I-V characteristics:

- junction voltage sets *concentration* of carriers with enough energy to get injected;
- rate of carrier injection set by minority carrier transport and G/R rates in quasi-neutral regions;
- SCR is in *quasi-equilibrium*.





Strategy for deriving first-order model for I-V characteristics:

- Compute diode current density as follows:

$$J = J_e(-x_p) + J_h(x_n)$$

- Compute each minority carrier current contribution as follows:

$$J_e(-x_p) = -qn'(-x_p)v_e(-x_p)$$

$$J_h(x_n) = qp'(x_n)v_h(x_n)$$

- Use expressions of $v_e(-x_p)$ and $v_h(x_n)$ derived for similar minority carrier type problems in Ch. 5.
- Derive expressions for $n'(-x_p)$ and $p'(x_n)$ assuming *quasi-equilibrium* across the space-charge region.
- Unified result for forward and reverse bias.

★ Boundary conditions across SCR.

In thermal equilibrium, Boltzmann relations:

$$\phi(x_n) - \phi(-x_p) = \phi_{bi} = \frac{kT}{q} \ln \frac{n_o(x_n)}{n_o(-x_p)}$$

$$\phi(x_n) - \phi(-x_p) = \phi_{bi} = -\frac{kT}{q} \ln \frac{p_o(x_n)}{p_o(-x_p)}$$

If net current inside SCR is much smaller than drift and diffusion components, then *quasi-equilibrium* \Rightarrow Boltzmann relations apply:

$$\phi_{bi} - V \simeq \frac{kT}{q} \ln \frac{n(x_n)}{n(-x_p)}$$

$$\phi_{bi} - V \simeq -\frac{kT}{q} \ln \frac{p(x_n)}{p(-x_p)}$$

In LLI, $n(x_n) \simeq N_D$, and $p(-x_p) \simeq N_A$. Also $\phi_{bi} = \frac{kT}{q} \ln \frac{N_D N_A}{n_i^2}$.
Then:

$$n(-x_p) \simeq \frac{n_i^2}{N_A} \exp \frac{qV}{kT}$$

$$p(x_n) \simeq \frac{n_i^2}{N_D} \exp \frac{qV}{kT}$$

In terms of excesses:

$$n'(-x_p) \simeq \frac{n_i^2}{N_A} \left(\exp \frac{qV}{kT} - 1 \right)$$

$$p'(x_n) \simeq \frac{n_i^2}{N_D} \left(\exp \frac{qV}{kT} - 1 \right)$$

Boundary conditions have all expected features. For electrons (for example):

- For $V = 0$:

$$n'(-x_p) = 0$$

- For $V \gg \frac{kT}{q}$:

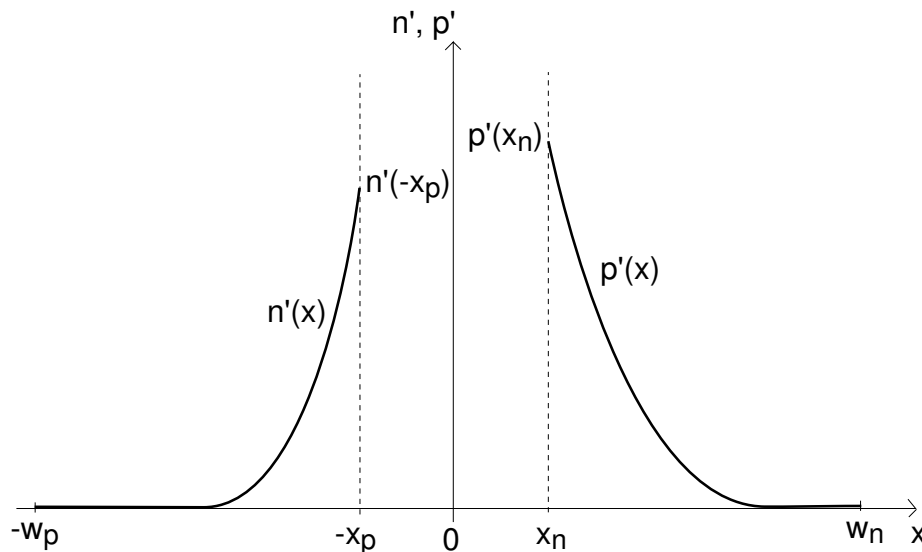
$$n'(-x_p) \simeq \frac{n_i^2}{N_A} \exp \frac{qV}{kT}$$

- For $V \ll -\frac{kT}{q}$:

$$n'(-x_p) \simeq -\frac{n_i^2}{N_A}$$

★ Minority carrier velocity at edges of SCR ("long" diode: $W_n \gg L_h, W_p \gg L_e$).

Problem identical to "long bar" studied in Ch. 5: exponentially decaying excess minority carrier profiles.



$$n'(x) = n'(-x_p) \exp \frac{x + x_p}{L_e} \quad \text{in p-QNR: } x \leq -x_p$$

$$p'(x) = p'(x_n) \exp \frac{-x + x_n}{L_h} \quad \text{in n-QNR: } x \geq x_n$$

Carrier velocities:

$$v_e(-x_p) = v_e^{diff}(-x_p) = -\frac{D_e}{L_e}$$

$$v_h(x_n) = v_h^{diff}(x_n) = \frac{D_h}{L_h}$$

★ Excess minority carrier currents:

$$J_e(-x_p) \simeq -qv_e^{diff}(-x_p) n'(-x_p) = q \frac{D_e}{L_e} \frac{n_i^2}{N_A} \left(\exp \frac{qV}{kT} - 1 \right)$$

$$J_h(x_n) \simeq qv_h^{diff}(x_n) p'(x_n) = q \frac{D_h}{L_h} \frac{n_i^2}{N_D} \left(\exp \frac{qV}{kT} - 1 \right)$$

★ Total current: sum of electron and hole current:

$$J = J_e(-x_p) + J_h(x_n) = qn_i^2 \left(\frac{1}{N_A} \frac{D_e}{L_e} + \frac{1}{N_D} \frac{D_h}{L_h} \right) \left(\exp \frac{qV}{kT} - 1 \right)$$

Define $J_s \equiv$ saturation current density (A/cm^2):

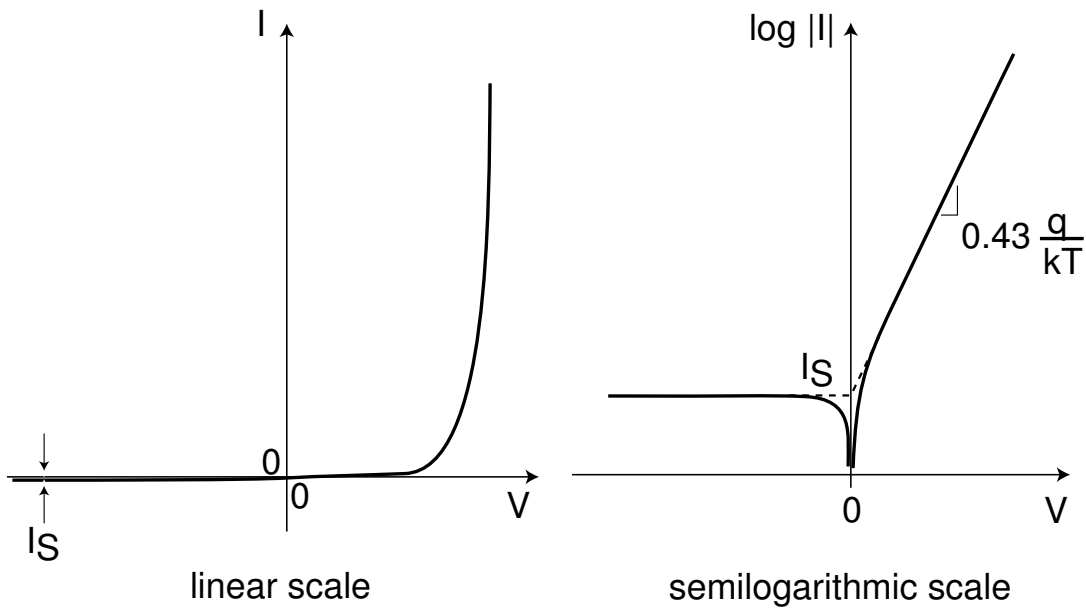
$$J = J_s \left(\exp \frac{qV}{kT} - 1 \right)$$

If diode area is A , current is:

$$I = I_s \left(\exp \frac{qV}{kT} - 1 \right)$$

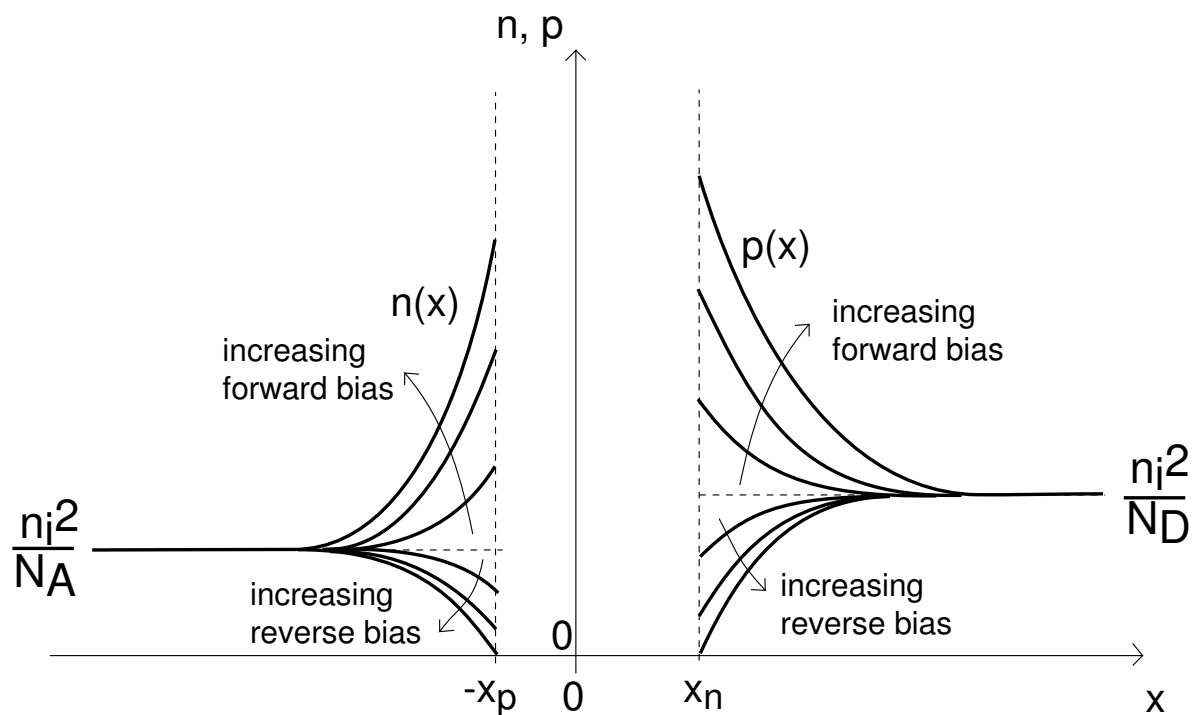
$$I = I_s \left(\exp \frac{qV}{kT} - 1 \right)$$

Classic rectifying behavior



Should test quality of quasi-equilibrium approximation [problem #7.13].

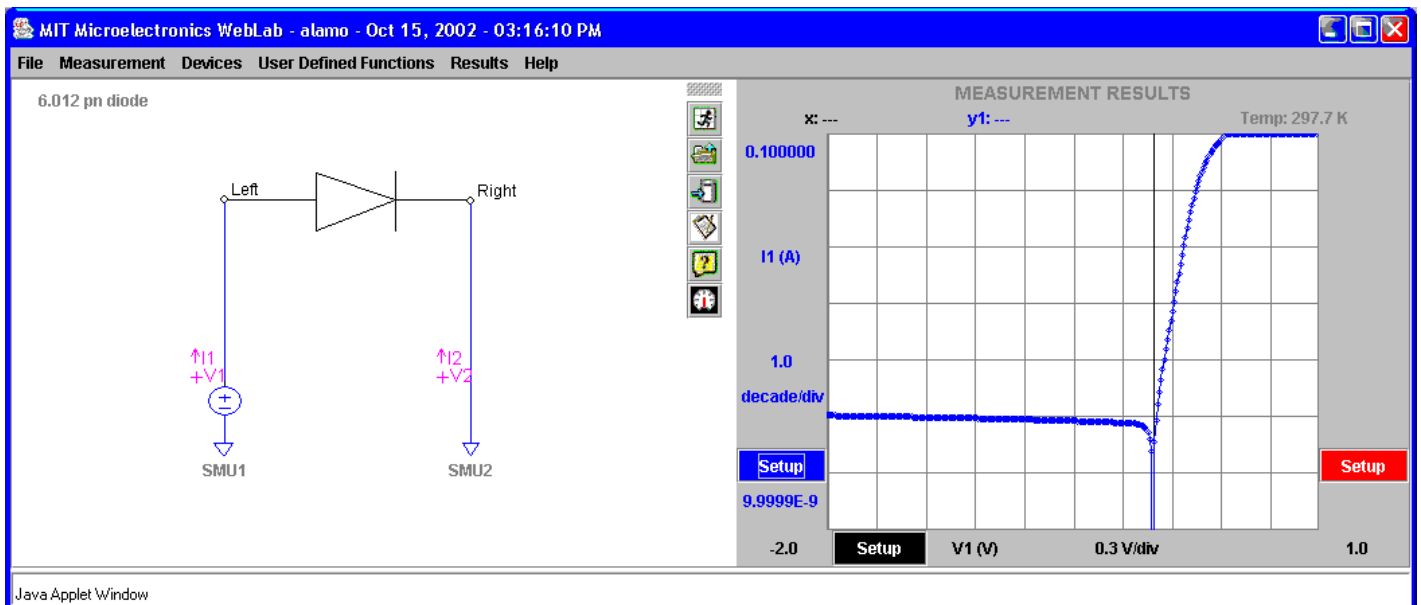
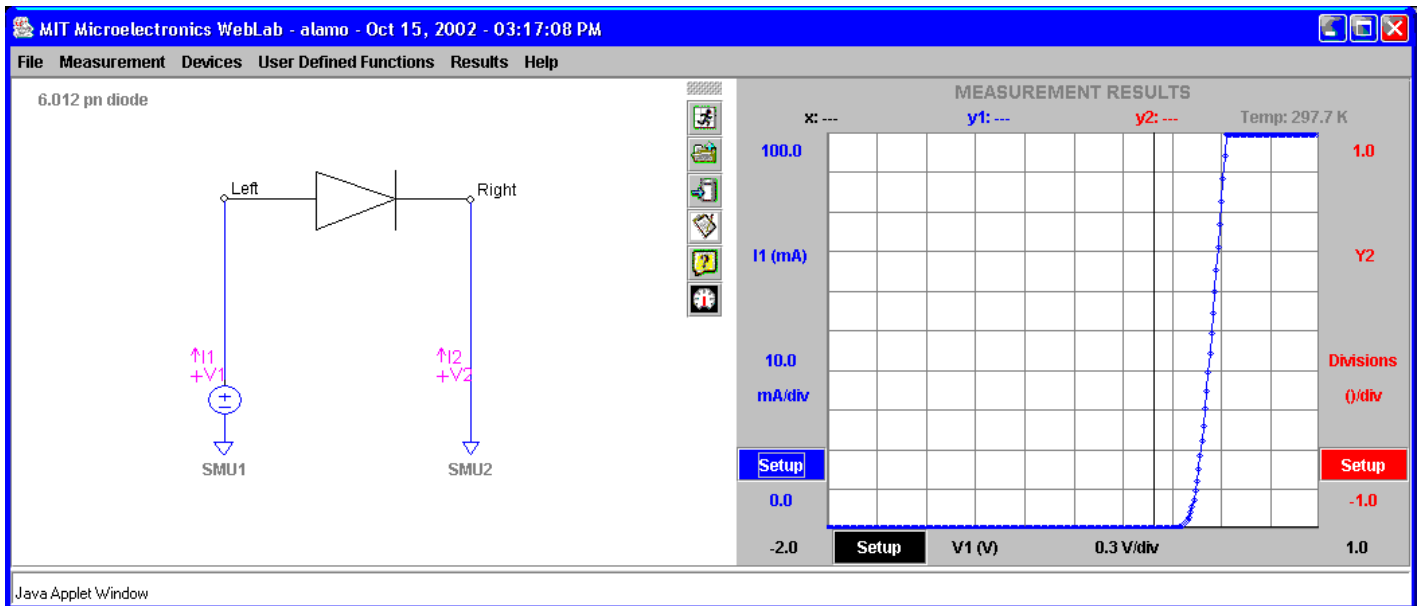
Rectifying behavior arises from boundary conditions across SCR:



-In forward bias: carrier concentrations at SCR edges grow up exponentially $\rightarrow I \sim e^{qV/kT}$

-In reverse bias: carrier concentrations at SCR edges reduced quickly to zero (can't go below!) $\rightarrow I$ saturates

Experimental verification:

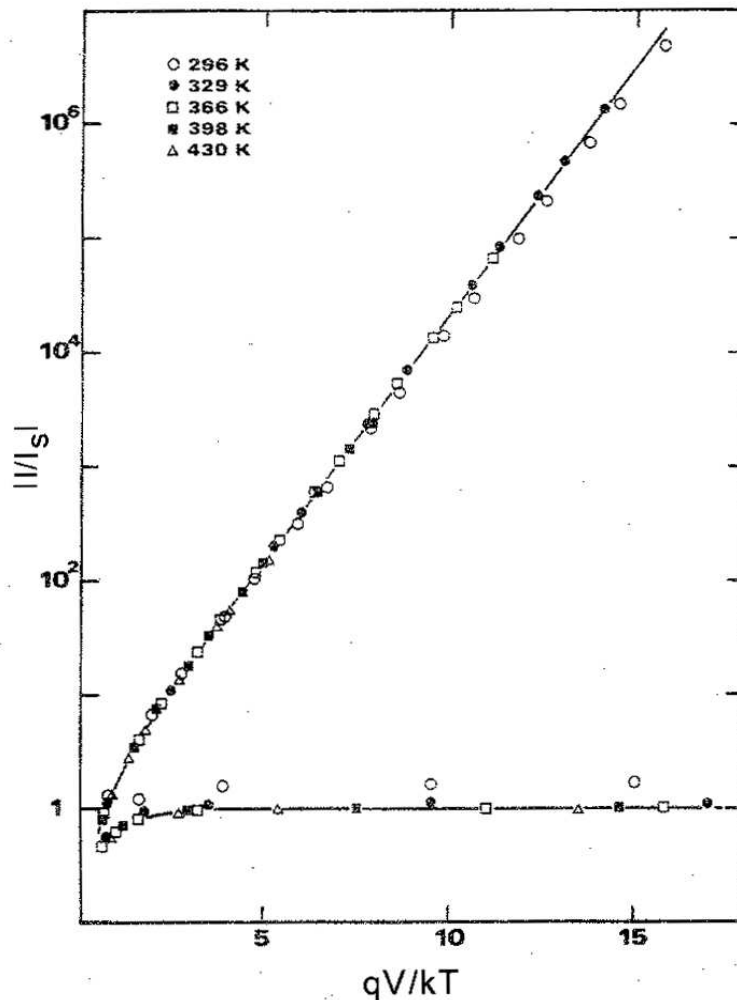


Universality of exponential relationship:

$$I = I_S \left(\exp \frac{qV}{kT} - 1 \right)$$

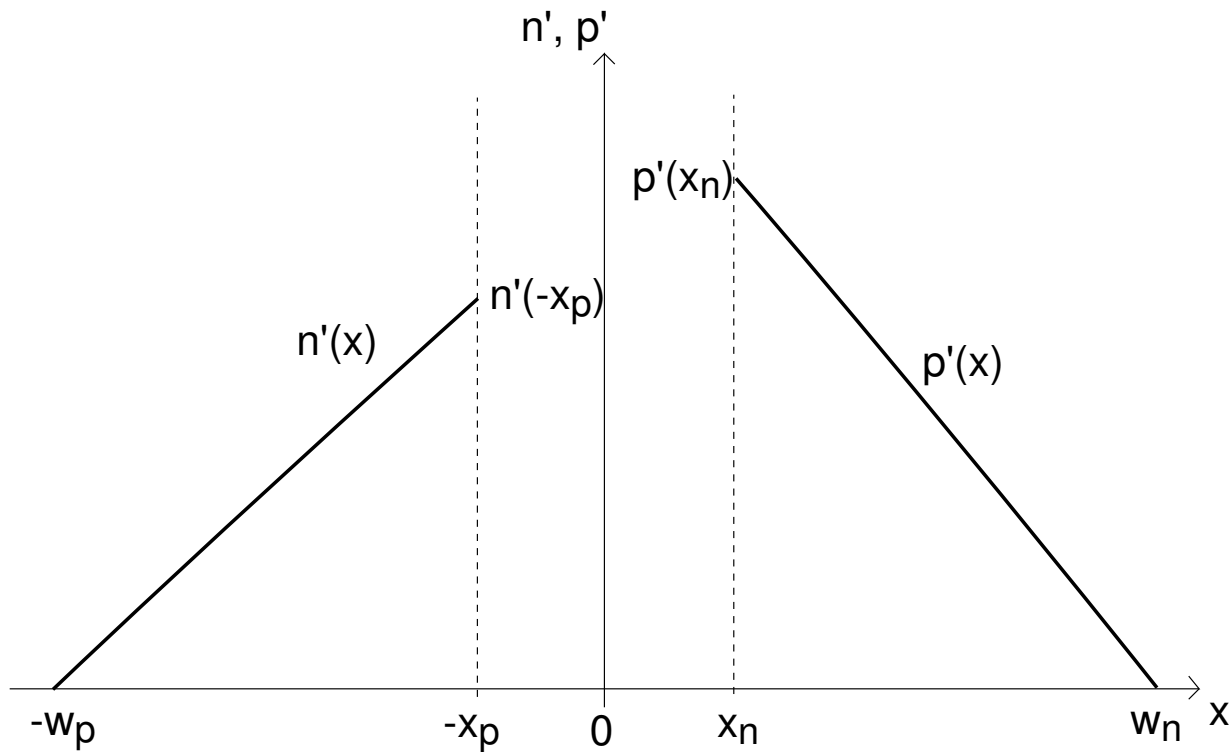
Then:

$$\frac{I}{I_S} = \exp \frac{qV}{kT} - 1$$



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In short diodes with $S = \infty$, G&R takes place at surfaces:



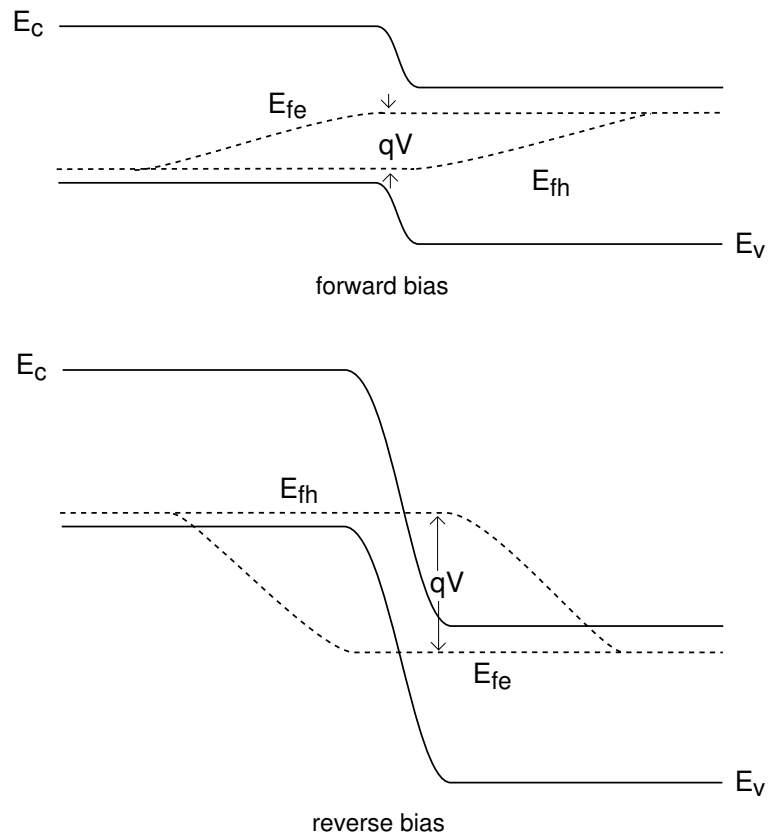
$$v_e^{diff}(-x_p) = -\frac{D_e}{w_p - x_p}$$

$$v_h^{diff}(x_n) = \frac{D_h}{w_n - x_n}$$

J_s is:

$$J_s \simeq qn_i^2 \left(\frac{1}{N_A} \frac{D_e}{w_p - x_p} + \frac{1}{N_D} \frac{D_h}{w_n - x_n} \right)$$

Quasi-Fermi levels across long diode:



Inside SCR:

$$E_{fe} - E_{fh} = qV$$

Then:

$$np = n_i^2 \exp \frac{qV}{kT}$$

From here can get also BC's.

Key conclusions

- Forward bias: junction barrier $\downarrow \Rightarrow$ carrier injection \Rightarrow recombination in QNR's $\Rightarrow I \sim e^{qV/kT}$
- Reverse bias: junction barrier $\uparrow \Rightarrow$ carrier extraction \Rightarrow generation in QNR's $\Rightarrow I$ saturates with reverse V
- Rectifying characteristics of pn diode arise from boundary conditions at edges of SCR.
- Excess minority carrier concentration at edges of depletion region:

$$n'(-x_p) = \frac{n_i^2}{N_A} \left(\exp \frac{qV}{kT} - 1 \right), \quad p'(x_n) = \frac{n_i^2}{N_D} \left(\exp \frac{qV}{kT} - 1 \right)$$

- I-V characteristics of ideal pn diode:

$$I = I_s \left(\exp \frac{qV}{kT} - 1 \right)$$

- Quasi-Fermi levels flat across SCR:

$$np = n_i^2 \exp \frac{qV}{kT} \quad \text{inside SCR}$$