Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 4 Solution

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Problem 1: No secrets here. The script to do the vee curve is appended. The script, first, finds the field winding capability, armature winding capability and then finds the stability limit (using a slightly different method from Problem Set 3). The torque angle is used as a parameter. Note the limits for field winding and armature currents are checked in a straightforward fashion and noted on the plot. The plot is shown in Figure 1. Note the problem statement is flawed: there is no vee curve for the 1,000 MW curve, which is right on the armature capacity and therefore is a single point.
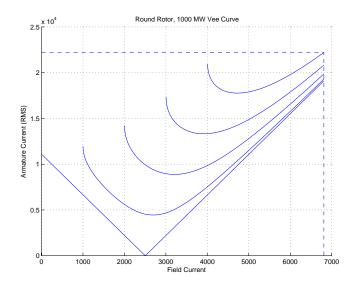

Figure 1: Vee Curve for Example Generator

Problem 2: Salient Pole Machine

1. To start, note that this machine will have a stability limit for operation at low field excitation (corresponding to high absorbed reactive power). For a round rotor machine this limit is reached at a torque angle of 90°, but this machine has saliency so you must determine the value of angle for which stability is reached. Compute and plot the angle and corresponding value of field current at the stability threshold for this machine, against real power. *Hint:* The stability limit is reached when the derivative of torque with respect to angle is zero. Since torque is proportional to real power, you can use the derivative of power with angle. For this part of the problem, ignore resistances and core loss.

1

Real power output for a generator is:

$$P = 3\frac{V E_{af}}{X_d} \sin \delta + \frac{3}{2}V^2 \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta$$

The derivative of power with angle is then simply:

$$\frac{dP}{d\delta} = 3\frac{V E_{af}}{X_d} \cos \delta + 3V^2 \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \cos 2\delta$$

At the stability limit, $\frac{dP}{d\delta} = 0$, and this may be solved for internal voltage:

$$E_{af} = -V \left(\frac{X_d}{X_q} - 1 \right) \frac{\cos 2\delta}{\cos \delta}$$

Using this shorthand:

$$P_0 = 3V^2 \left(\frac{1}{X_q} - \frac{1}{X_d} \right)$$

we have this nonlinear expression to solve:

$$\frac{P}{P_0} \cos \delta - \frac{1}{2} \sin 2\delta + \cos 2\delta \sin \delta = 0$$

Now, this looks awful but in fact is quite easily solved by most mathematical assistants. MATLAB, for example, has a routine called 'fzero' which makes quick work of it. Once δ is found, E_{af} may be determined and the operating point is easily determined.

2. Reactive power at the underexcited stability limit is plotted in Figure 2 Note also the values of field current and torque angle in Figure 3.

3. Calculation of required field current and calculation of the torque-angle curve is carried out in the third script appended.

For this, see Figure 4

4. Calculation of efficiency is fairly straightforward. Take the notion that efficiency is:

$$\eta = \frac{P}{P + P_a + P_f + P_c}$$

The calculation is carried out in the normal fashion, assuming that, for unity power factor:

$$
\begin{aligned}
I_a &= \frac{P}{3V} \\
E_1 &= V + jX_q I_a \\
\delta &= \text{angle}(E_1) \\
I_d &= I_a \sin \delta \\
E_{af} &= abs(E_1) + (X_d - X_q)I_d \\
I_f &= I_{fnl}\frac{E_{af}}{V} \\
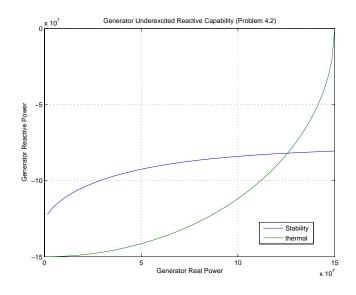P_a &= 3R_a I_a^2 \\
P_f &= R_f I_f^2
\end{aligned}
$$

Figure 2: Reactive and Thermal Reactive Power Limit
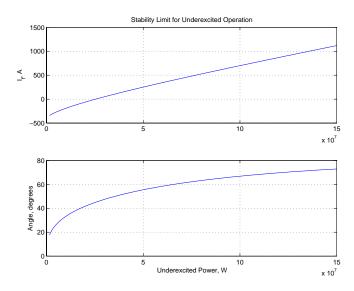
The results are shown in Figure 5
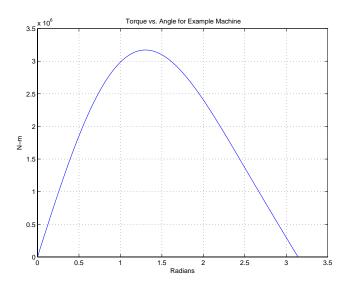
Figure 3: Underexcited Stability Limit

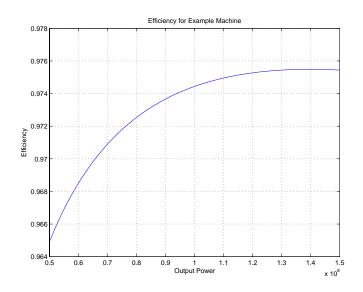Figure 4: Torque-Angle Curve for Example Machine (Generator Coordinates)

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Figure 5: Example Machine Efficiency

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Problem 3: This is more scutwork than one might like. Note that calculation of the *phase* inductance matrix is:

$$\underline{\underline{L}}_{ph} = \underline{\underline{T}}^{-1}\underline{\underline{L}}_{dq}\underline{\underline{T}}$$

So the second part of that is:

$$
\underline{\underline{L}}_{dq}\underline{\underline{T}} = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_0 \end{bmatrix} \frac{2}{3} \begin{bmatrix} \cos\phi & \cos(\phi - \frac{2\pi}{3}) & \cos(\phi + \frac{2\pi}{3}) \\ -\sin\phi & \sin(\phi - \frac{2\pi}{3}) & -\sin(\phi + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}
$$

$$
= \frac{2}{3} \begin{bmatrix} L_d\cos\phi & L_d\cos(\phi - \frac{2\pi}{3}) & L_d\cos(\phi + \frac{2\pi}{3}) \\ -L_q\sin\phi & -L_q\sin(\phi - \frac{2\pi}{3}) & -L_q\sin(\phi + \frac{2\pi}{3}) \\ \frac{L_0}{2} & \frac{L_0}{2} & \frac{L_0}{2} \end{bmatrix}
$$

Then

$$
\underline{\underline{L}}_{ph} = \begin{bmatrix} L_a & L_{ab} & L_{ac} \\ L_{ab} & L_b & L_{bc} \\ L_{ac} & L_{bc} & L_c \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos\phi & -\sin\phi & 1 \\ \cos(\phi - \frac{2\pi}{3}) & \sin(\phi - \frac{2\pi}{3}) & 1 \\ \cos(\phi + \frac{2\pi}{3}) & -\sin(\phi + \frac{2\pi}{3}) & 1 \end{bmatrix} \begin{bmatrix} L_d\cos\phi & L_d\cos(\phi - \frac{2\pi}{3}) & L_d\cos(\phi + \frac{2\pi}{3}) \\ -L_q\sin\phi & -L_q\sin(\phi - \frac{2\pi}{3}) & -L_q\sin(\phi + \frac{2\pi}{3}) \\ \frac{L_0}{2} & \frac{L_0}{2} & \frac{L_0}{2} \end{bmatrix}
$$

Carrying out this last operation, one finds a few elements of the answer:

$$
L_a = \frac{2}{3}\left(L_d\cos^2\phi + L_q\sin^2\phi + \frac{L_0}{2}\right)
$$

$$
L_{ab} = \frac{2}{3}\left(L_d\cos\phi\cos(\phi - \frac{2\pi}{3}) + L_q\sin\phi\sin(\phi - \frac{2\pi}{3}) + \frac{L_0}{2}\right)
$$

Doing the trig, one finds:

$$
L_a = \frac{1}{3}\left((L_d + L_q) + (L_d - L_q)\cos(2\phi) + L_0\right)
$$

$$
L_b = \frac{1}{3}\left((L_d + L_q) + (L_d - L_q)\cos 2(\phi - \frac{2\pi}{3}) + L_0\right)
$$

$$
L_c = \frac{1}{3}\left((L_d + L_q) + (L_d - L_q)\cos 2(\phi + \frac{2\pi}{3}) + L_0\right)
$$

$$
L_{ab} = -\frac{1}{6}(L_d + L_q) + \frac{1}{6}(L_d - L_q)\cos(2\phi - \frac{2\pi}{3}) + \frac{1}{3}L_0
$$

$$
L_{ac} = -\frac{1}{6}(L_d + L_q) + \frac{1}{6}(L_d - L_q)\cos(2\phi + \frac{2\pi}{3}) + \frac{1}{3}L_0
$$

$$
L_{bc} = -\frac{1}{6}(L_d + L_q) + \frac{1}{6}(L_d - L_q)\cos 2\phi + \frac{1}{3}L_0
$$

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```
% Round-Rotor Machine Vee Curve Generator
% Assumes generator operation
% Machine Data First
om=2*pi*60;                 % this is a 60 Hz machine
VA = 1e9;                   % armature rating
Xd = 1.353;                 % synchronous reactance (ohms)
V = sqrt(2/3)*26000;        % voltage: peak phase voltage
pfr = .8;                   % rating point
M = 22.5e-3;                % field-phase mutual reactance
Pv = [200e6 400e6 600e6 800e6];
% find armature current capability
Iar = VA/(1.5 * V);         % remember we are working in peak

% find maximum field capability:

psir = acos(pfr);           % power factor angle
% Ia is the complex current at rating point
Ia = Iar*cos(psir) + j*Iar*sin(psir);
Eafr = V - j*Xd*Ia;
Eafm = abs(Eafr);           % this is max magnitude of field voltage
Ifm = Eafm/(om*M);

fprintf('Rated Ia = %g  Max If = %g\n',Iar, Ifm)
figure(1)
clf
hold on

for i = 1:length(Pv)
    P = Pv(i);
    Q = sqrt(VA^2-P^2);
    Ia_min = (P+j*Q)/(1.5*V);           % under-excited: check for stability
    Eaf_min = V + j*Xd*Ia_min;
    if angle(Eaf_min) > pi/2,
                    dmin =  pi/2;
        else
                    dmin = angle(Eaf_min);
    end
    Ia_max = (P-j*Q)/(1.5*V);               % over-excited: check for Ifmax
    Eaf_max = V+j*Xd*Ia_max;
    if abs(Eaf_max) < Eafm,
                  dmax = angle(Eaf_max);
    else
                  dmax = asin(P*Xd/(1.5*V*Eafm));
    end
    fprintf('P= %g Ia_min = %g Ia_max = %g dmin = %g  dmax = %g\n', P, abs(Ia_min), abs(Ia_ma:
```

```matlab
% now that we have limits we go parameterize by delta

    delt = dmin:(dmax-dmin)/100:dmax;
    Eaf = P*Xd ./ (1.5*V .* sin(delt));
    Q = 1.5*(V^2/Xd - (V/Xd) .* Eaf .* cos(delt));
    Ia = sqrt(P^2 + Q .^2)/(1.5*V);
    If = Eaf ./ (om*M);

    plot(If, Ia ./ sqrt(2))
end


% now for the zero power curve
    Iaz = [V/Xd 0 (Eafm-V)/Xd];
    Ifz = [0 V/(om*M) Eafm/(om*M)];
plot(Ifz, Iaz ./ sqrt(2))

% and now we plot the limit lines
    Ial = [Iar Iar 0];
    Ifl = [0 Ifm Ifm]
plot(Ifl, Ial ./ sqrt(2), '--')

title('Round Rotor, 1000 MW Vee Curve')
ylabel('Armature Current (RMS)')
xlabel('Field Current')
grid on
hold off
```

```
% 6.685 2013 Problem Set 4, Problem 2

% First, line up parameters
VA = 150e6;                      % Machine Rating
V = 13800/sqrt(3);           % Phase voltage, RMS
om = 2*pi*60;                   % frequency
Xd = 2.5;                % direct axis reactance
Xq = 1.5;                % quadrature axis reactance
AFNL = 600;                    % rated voltage open circuit field current
p = 7;

% Find Stability Limit for underexcited operation
% This will be angle delta as a function of real power P
% warning off MATLAB:fzero:UndeterminedSyntax    % to suppress a whole lot of wierd warnings
P_0 = 3*V^2*(1/Xq-1/Xd);        % convenient shorthand
P = (.01:.01:1) .* VA;          % establish a range of real power
Qs = zeros(size(P));         % space for Q
Qc = zeros(size(P));
E_af = zeros(size(P));
ds = zeros(size(P));

for i = 1:length(P)
    Pr = P(i)/P_0;              % here is how we use the notation
    d = fzero('ef', [0 pi/2], [], Pr);  % this gives angle at stability limit
    Eaf = -V*(Xd/Xq-1)*cos(2*d)/cos(d); % and corresponding internal voltage
    E_af(i) = Eaf;
    ds(i) = d;
    Qs(i) = (3*V*Eaf/Xd) * cos(d) + 1.5*V^2*(1/Xq-1/Xd) * cos(2*d) - 1.5*V^2*(1/Xq+1/Xd);
    Qc(i) = -sqrt(VA^2-P(i)^2);
end
dpdd = (3*V/Xd) .* E_af .* cos(ds) +3*V^2*(1/Xq-1/Xd) .* cos(2 .* ds);


figure(1)
plot(P, Qs, P, Qc)
legend('Stability', 'thermal')
title('Generator Underexcited Reactive Capability (Problem 4.2)')
ylabel('Generator Reactive Power')
xlabel('Generator Real Power')
%axis([0 2e8 -1e8 0])
%axis square
grid on

I_f = (AFNL/V) .* E_af;
```

```
figure(2)
subplot 211
plot(P, I_f)
title('Stability Limit for Underexcited Operation')
ylabel('I_f, A')
grid on
subplot 212
plot(P, (180/pi) .* ds)
ylabel('Angle, degrees')
xlabel('Underexcited Power, W')
grid on
```

```
% 6.685 2013 Problem Set 4, Problem 2, parts 3 and 4

% First, line up parameters
VA = 150e6;                        % Machine Rating
V = 13800/sqrt(3);          % Phase voltage, RMS
om = 2*pi*60;                % frequency
Xd = 2.5;                  % direct axis reactance
Xq = 1.5;                 % quadrature axis reactance
AFNL = 600;                 % rated voltage open circuit field current
p = 7;                      % number of pole pairs
Ra = .009;                  % armature resistance
Rf = 1.0;                   % field resistance
Pc = 1e6;                   % core loss

% First, find field current for rated operation, unity power factor

Ia = VA/(3*V);                  % this would be current at rated operation
E_1 = V + j*Xq*Ia;          % spot on the q axis
delta = angle(E_1);       % this is torque angle
Id = Ia * sin(delta);     % d- axis current
Eaf = abs(E_1) + (Xd-Xq)*Id;   % internal voltage

I_f = Eaf*AFNL/V;

fprintf('Problem 4, part 3: Field Current = %g\n', I_f);
fprintf('Details of that: E_1 = %g, delta = %g\n', abs(E_1), delta);
fprintf('More details: I_d = %g, E_af = %g\n', Id, Eaf);

delt = 0:pi/100:pi;
P = (3*V*Eaf/Xd) .* sin(delt) + 1.5*V^2*(1/Xq - 1/Xd) .* sin(2 .* delt);

T = (p/om) .* P;

figure(1)
plot(delt, T)
title('Torque vs. Angle for Example Machine')
ylabel('N-m')
xlabel('Radians')
grid on

% now for machine efficiency

P = 50e6:5e5:150e6;        % run over this range
Ia = P ./ (3*V);           % armature current
E_1 = V + Xq .* Ia;        % voltage on q axis
```

```
delta = angle(E_1);        % torque angle
Id = Ia .* sin(delta);     % d- axis current
Eaf = E_1 + (Xd-Xq) .* Id;
I_f = (AFNL/V) .* Eaf;      % required field current

P_a = 3*Ra .* Ia .^2;      % armature winding loss
P_f = Rf .* I_f .^2;       % field winding loss

eff = P ./ (P + P_a + P_f + Pc);

figure(2)
plot(P, eff)
title('Efficiency for Example Machine')
ylabel('Efficiency')
xlabel('Output Power')
grid on
```

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