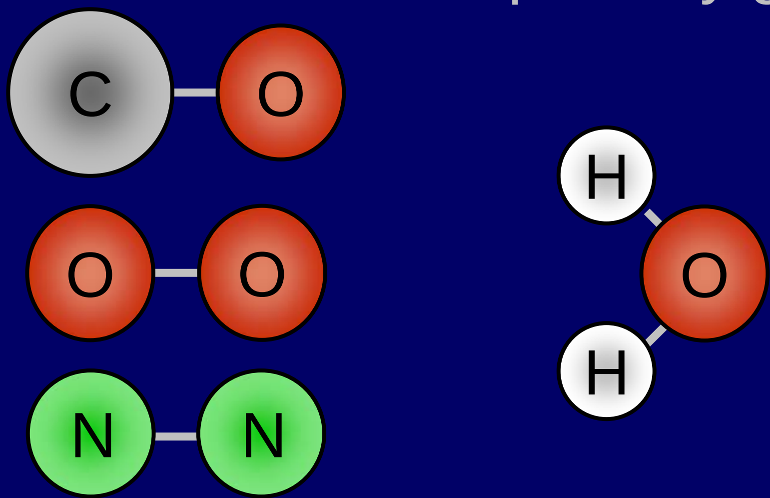
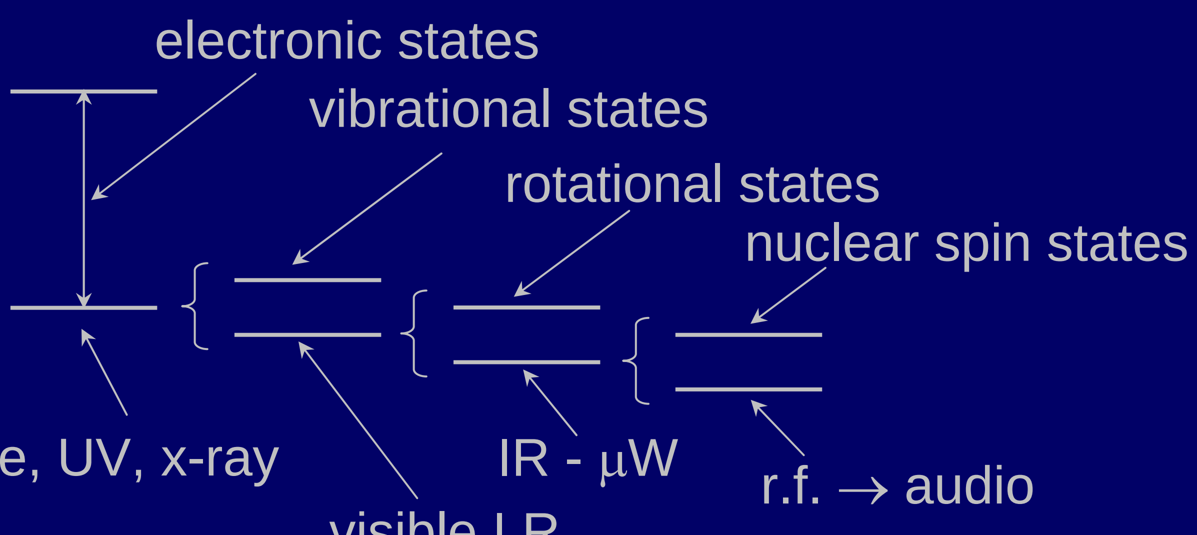
Wave Propagation

Molecular line absorption by gases:



- permanent electric dipole (H_2O, CO)
- permanent magnetic dipole (O_2)
- unpolarized (N_2) (collision-induced dipoles)

Quantized energy levels: $E_i - E_j = hf$



Molecular Lines in Gases

Quantized energy levels: $E_i - E_j = hf$

Probability of radiation = $A + B\,\rho_f$

"Einstein 'A' coeff."

"Einstein 'B' coeff."

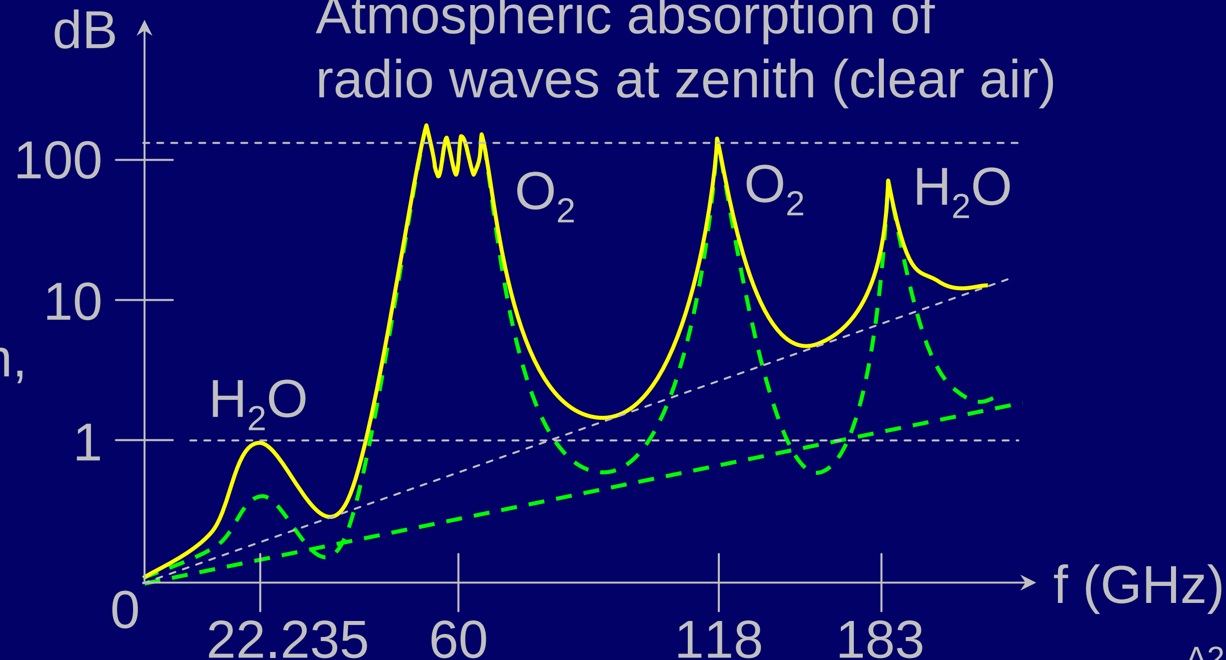
radiation intensity (energy density)

Probability of absorption = $B\rho_f$

$$A/B = 8\pi h f^3 / c^3$$

Collisions and radiation compete to control level populations. In equilibrium, kinetic and radiation temperatures are equal.

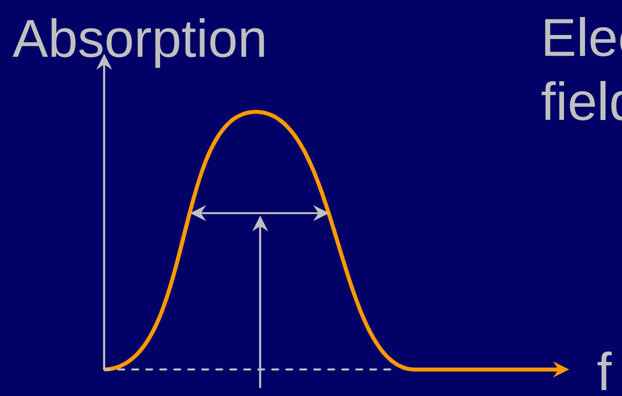
Atmospheric absorption of radio waves at zenith (clear air)

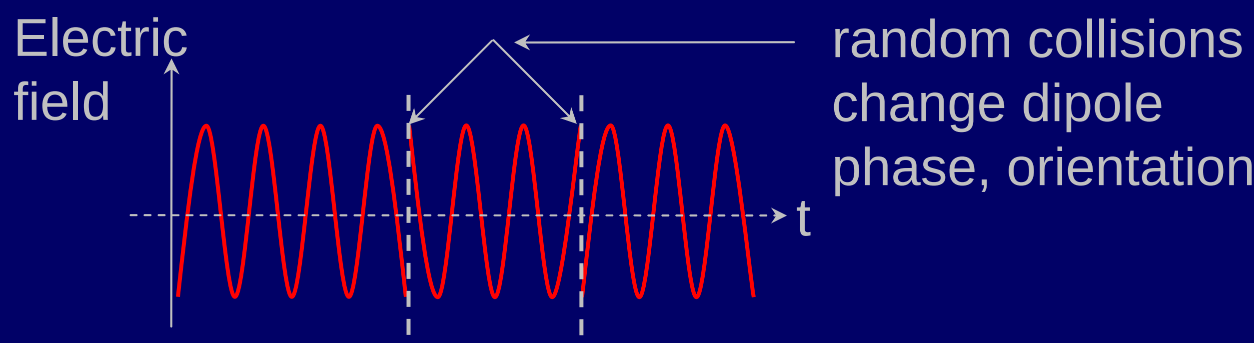


Molecular Line Shape

Broadening: intrinsic, collisional, Doppler

Einstein "A" yields spontaneous emission, limiting state lifetime T; intrinsic linewidth $\cong$ 1/T





$\Delta\omega$ = linewidth $\propto$ number of collisions/sec

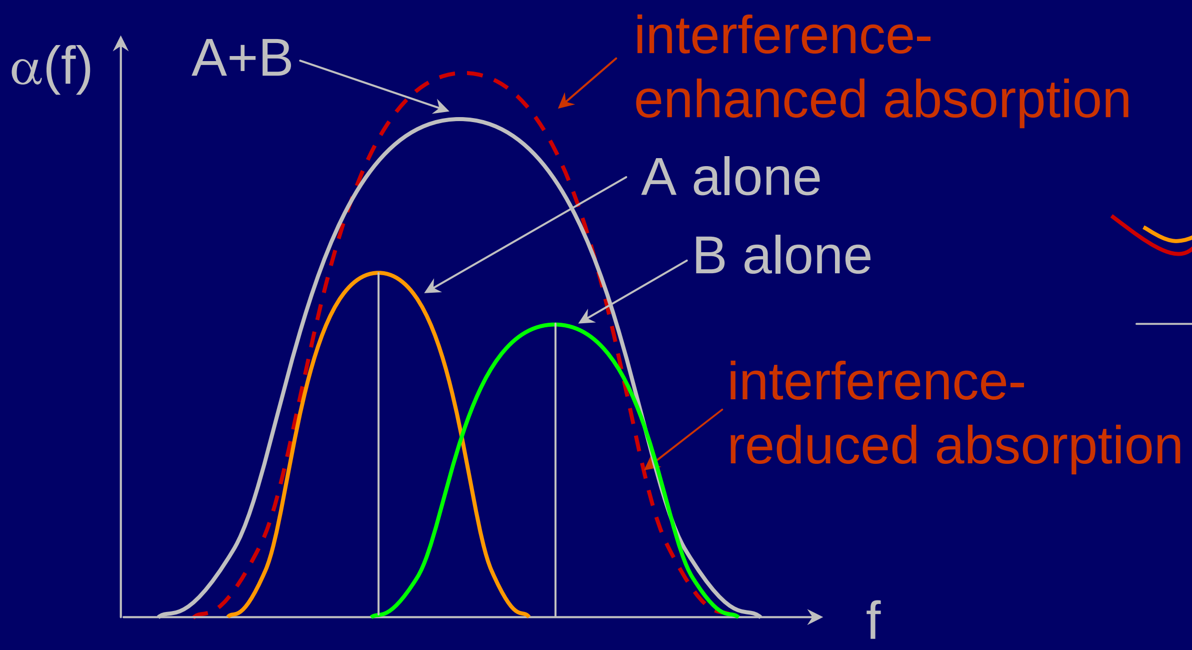
"Pressure broadening" or "collision broadening" $\cong$ 2 GHz at standard temperature and pressure (STP) for O_2, H_2O < 1 THz

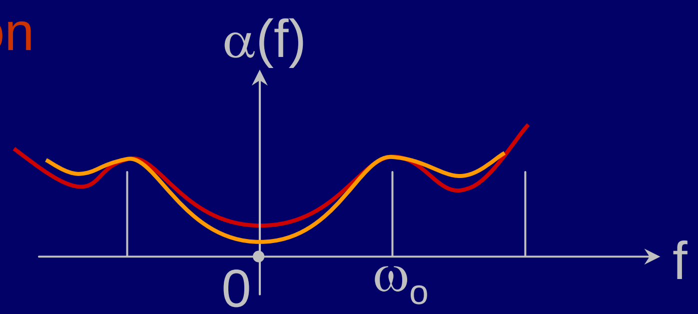
Doppler broadening has thermal $\left(\frac{1}{2}m\overline{v^2} \cong \frac{3}{2}kT\right)$, turbulent (random), and systematic components

Overlapping Spectral Lines

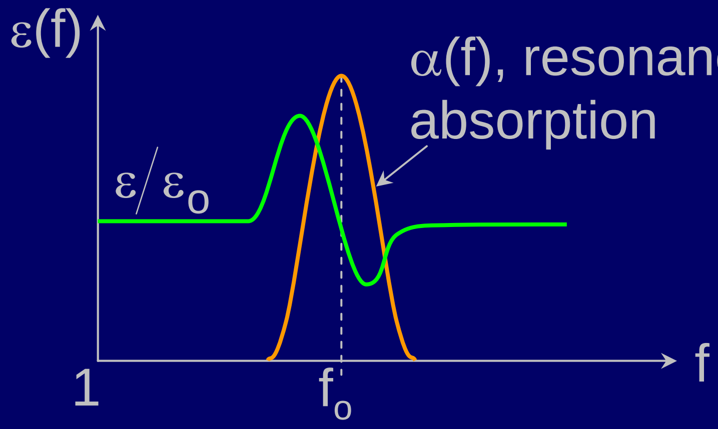
Superposition characterizes the cumulative absorption by independent spectral lines, except for certain single molecules.

For a single molecule with collision-coupled states, total absorption is generally greater than sum of line absorptions between coupled lines, and less outside. Such coupled lines coherently "interfere" (e.g. 60-GHz oxygen band).





Refractive Effects



The permittivity $\varepsilon(f)$ of a medium is related in part to the absorption coefficient $\alpha(f)$ by the Hilbert transform; $\alpha(f)$ is related to the imaginary part of $\underline{\varepsilon}(f)$.

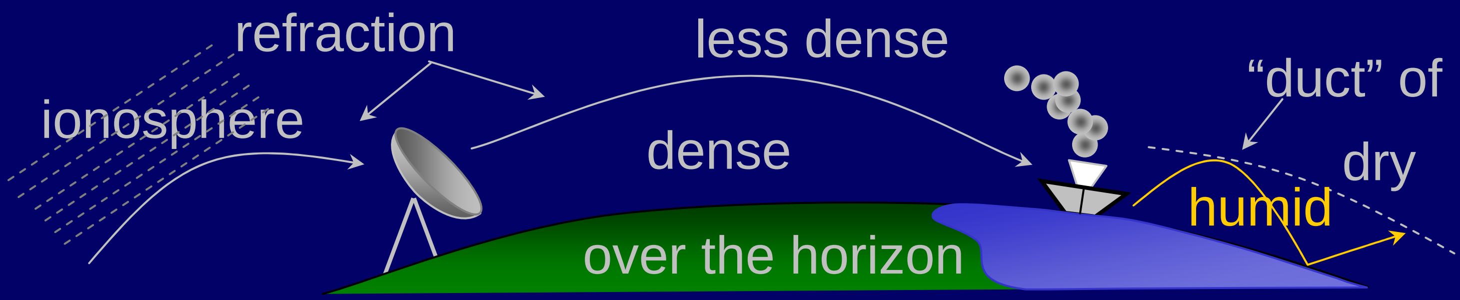
Atmospheric water vapor scale height = ~2 km
Atmospheric density scale height $\cong$ ~8 km
So humidity-based refractive effects are mostly a lower tropospheric phenomenon ($\lesssim$ 8 km).

Thermal inhomogeneities are turbulent in the boundary layer (first few hundred meters or more) and near convective instabilities, and are more layered at higher altitudes.

Humidity variations often dominate radio refraction, while density variations dominate optical propagation.
Optical telescopes have ~1 arc-sec “seeing” on good nights (2” – 10” in Boston is typical); the best mountain days may yield ~0.4 arc sec., where “seeing” is the blur spot size, not absolute refraction.

Refractive Effects

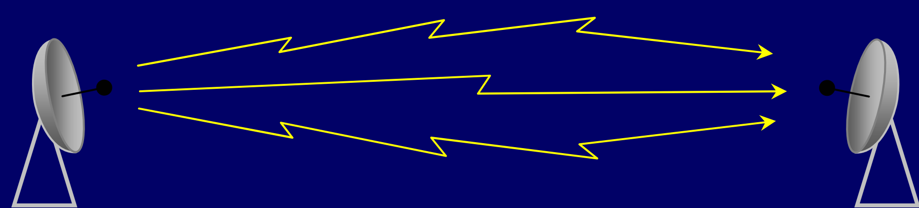
The radio index of refraction n is given by: $(n-1)10^6 = (79/T)(p + 4800\ e/T)$
where T is °K, and p and e are total and partial water vapor pressures (mb).



Ducting can occur in cold or humid layers of air, or in under-ionized ionospheric layers. Acoustic ducting can occur in cool or salty ocean layers.

Refractive seeing beyond the horizon can be $\gtrsim$ 30 arc minutes on RF, and less at optical frequencies.

Fading caused by interfering multipath: paths of different length cause different frequencies to cancel out or "fade."



Ionospheric and Space Plasmas

Plasmas can have both neutral and ionized components.

The ionosphere has $n_e \cong 10^7 - 10^{12} \left(m^{-3}\right)$ from ~50 to 5000 km altitude. Electron density n_e(max) is ~100 - 400 km.

Plasma frequency:

$$\omega_p = \sqrt{\frac{n_e q^2}{m\varepsilon_o}} \left(rs^{-1}\right) \text{ where } m = \frac{m_e m_i}{m_e + m_i} \cong m_e$$

$$\varepsilon = \varepsilon_o \left(1 - \omega_p^2 / \omega^2\right)$$

q = electron charge

Evanescent waves only, if $\omega < \omega_p$

Propagation delay:

$$\left.\begin{array}{ll} \text{phase velocity} & v_p = c / \sqrt{1 - (\omega_p / \omega)^2} > c \\ \text{group velocity} & v_g = c\sqrt{1 - (\omega_p / \omega)^2} < c \end{array}\right\} \begin{array}{l} \text{for } f > f_p \\ \left(v_g v_p = c^2 \text{ here}\right) \end{array}$$

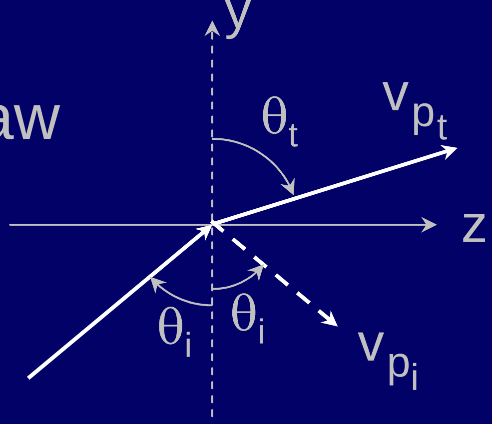
Refraction and Absorption by Plasmas

Refraction by plasmas:

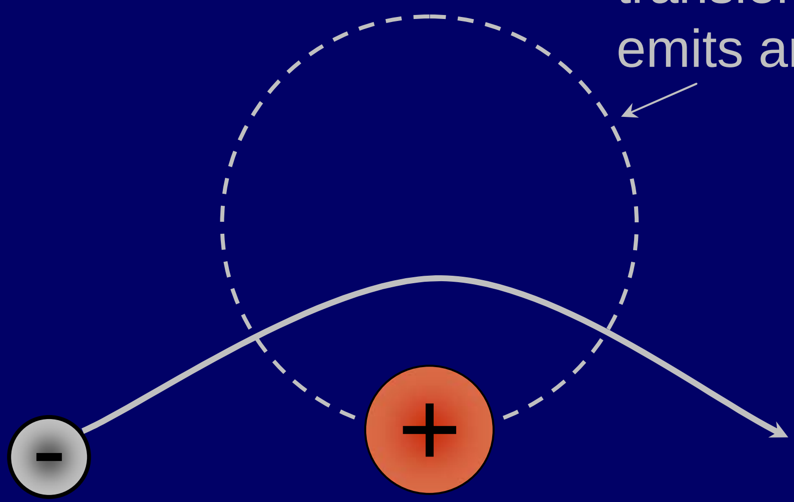
$\omega > \omega_p$ refraction is governed by Snell's Law

$$\sin\theta_i / \sin\theta_t = v_{p_t} / v_{p_i}$$

$\omega < \omega_p$ evanescent waves, total reflection



Absorption by plasmas:

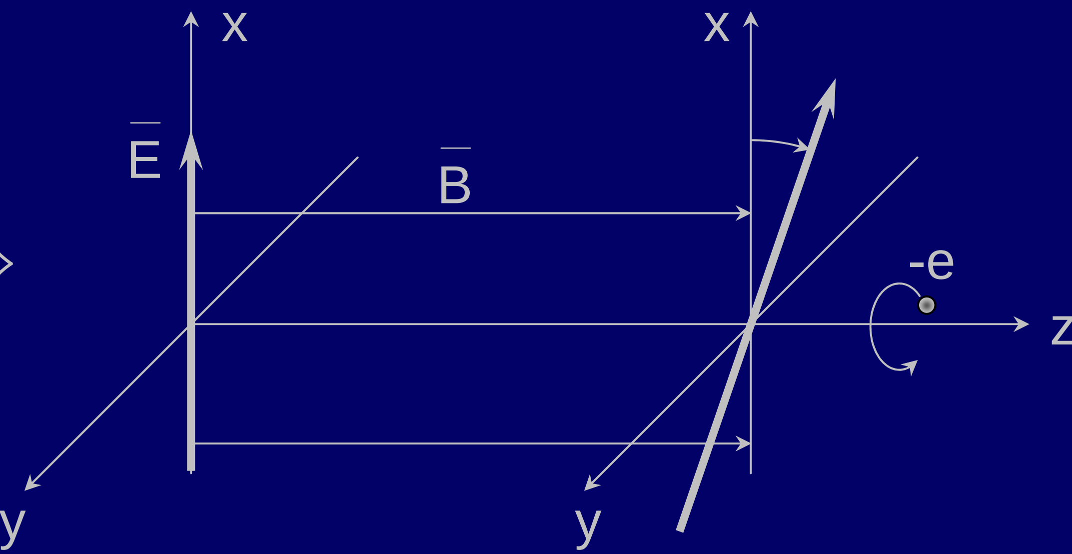


$\#$ collisions $\propto n_e^2$ (weak in ionosphere)

Magnetized Plasmas

Faraday rotation, bi-refringence

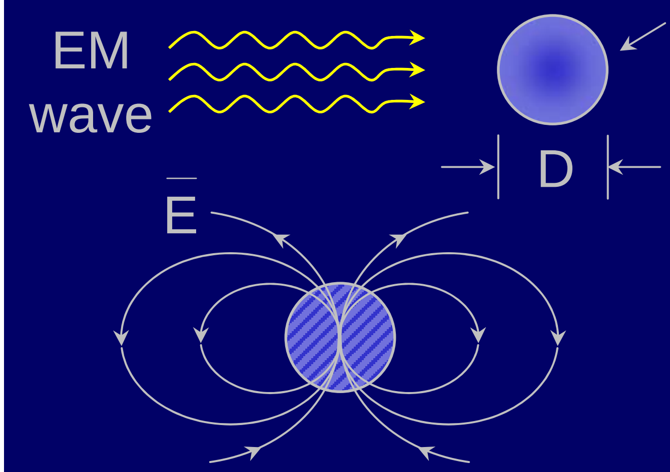
$$\overline{\overline{\varepsilon}} = \varepsilon_o \begin{bmatrix} K' & jK'' & 0 \\ -jK'' & K' & 0 \\ 0 & 0 & K_o \end{bmatrix} \Rightarrow$$



The EM interaction and Faraday rotation become strong near the electron and ion cyclotron resonances, $\omega_c = qB_o / m$

Scattering and Absorption by Dielectric Spheres

ε, sphere (e.g. raindrop)

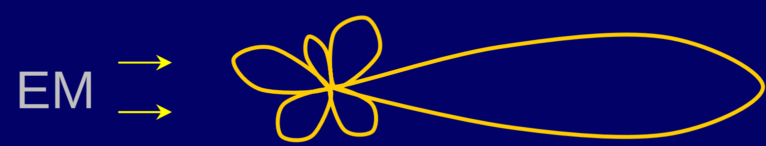


induced electric dipole

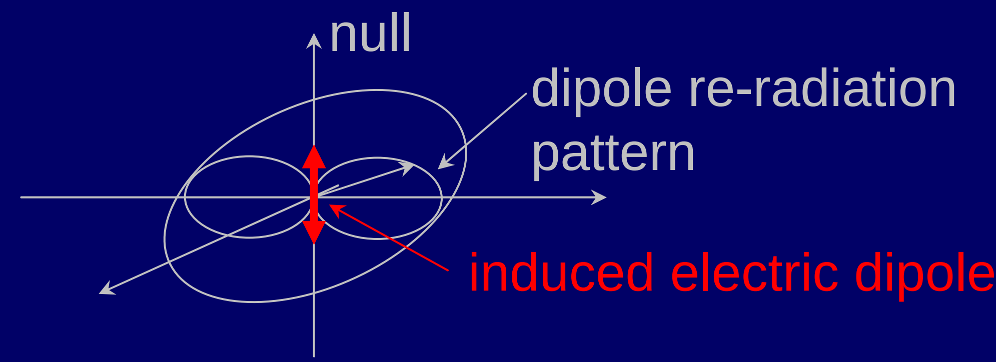
$$D << \frac{\lambda_o}{2\pi}\sqrt{\frac{\varepsilon_o}{\varepsilon}} \quad \Rightarrow \quad \text{"Rayleigh scattering"}$$

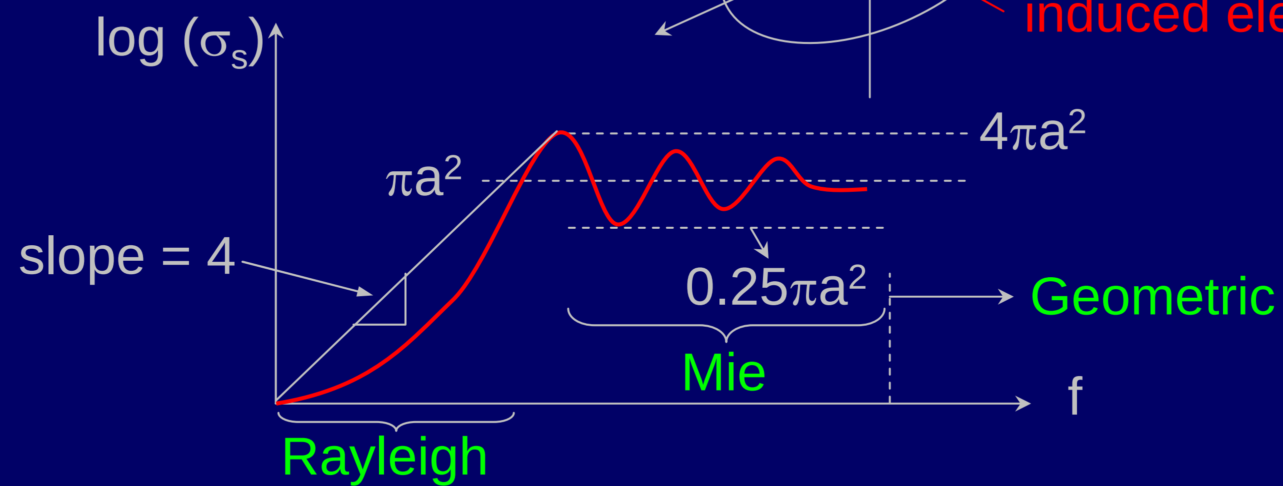
$$D \lesssim \frac{\lambda_o}{2\pi} \quad \Rightarrow \quad \text{"Mie scattering"}$$

$$D >> \frac{\lambda_o}{2\pi} \quad \Rightarrow \quad \text{Geometric scattering}$$



Mie is multimode:





Scattering and Absorption by Dielectric Spheres

Rayleigh regime $(\lambda >> 2\pi D\varepsilon/\varepsilon_o)$: (constant ε)

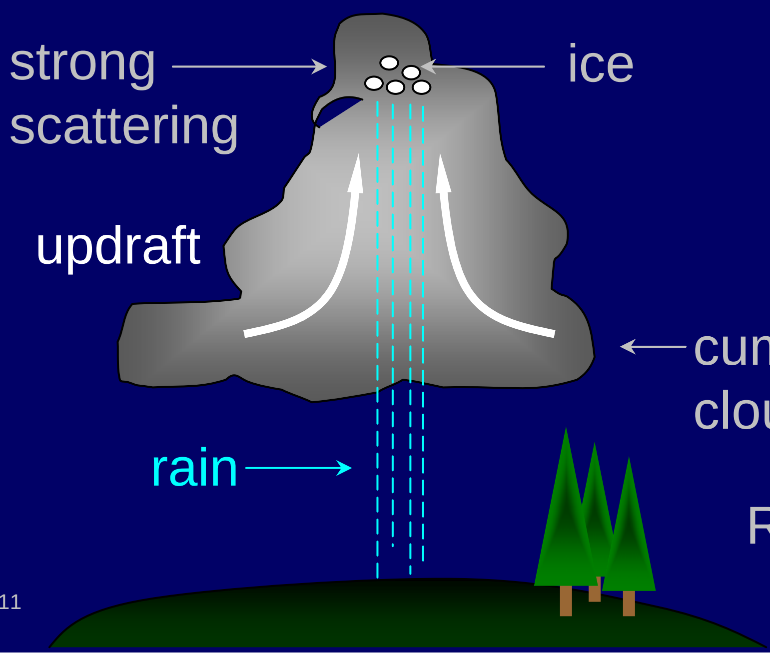
$$\sigma_s \propto (a/\lambda)^6 \lambda^2 \text{ scattering cross-section, } \alpha_{scattering}\left(\text{dBm}^{-1}\right) \propto f^4$$

$$\sigma_a \propto (a/\lambda)^3 \lambda \text{ absorption cross-section, } \alpha_{absorption}\left(\text{dBm}^{-1}\right) \propto f^2$$

Cloud absorption $\lesssim$ 85 GHz (Rayleigh region):

$$\gamma_{CLD}\left(\text{nepers cm}^{-1}\right) \cong \frac{m \bullet 10^{[0.0122(291-T)-6]}}{\lambda^2}$$

where m = g/m³ liquid water, λ = wavelength cm, T = K°



Albedo $\lesssim$ 0.8, $f_{max\ scat.} \approx 100 - 150$ GHz

$\therefore T_B \gtrsim$ 70K as seen from space

(Albedo $\stackrel{\Delta}{=}$ reflectivity, all angles)

Rain attenuation > 30dB sometimes