Amplitude Modulation "AM" with Envelope Detector

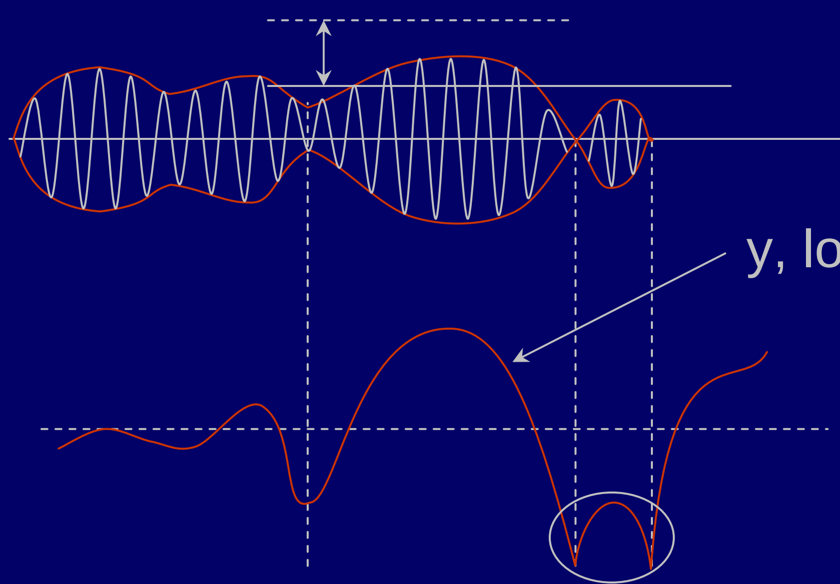
Large S/N limit

$m \lesssim 1 \qquad s(t) \cong \sin \omega_m t \le 1$

$$\text{Received} = y(t) = A_c\,[1 + m\,s(t)]\cos\omega_c t + n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$$

$$= \mathrm{Re}\left\{\underline{Y}(t)e^{j\omega_c t}\right\}$$

slowly varying



y, low-pass filtered (envelope)

"overmodulation" = distortion

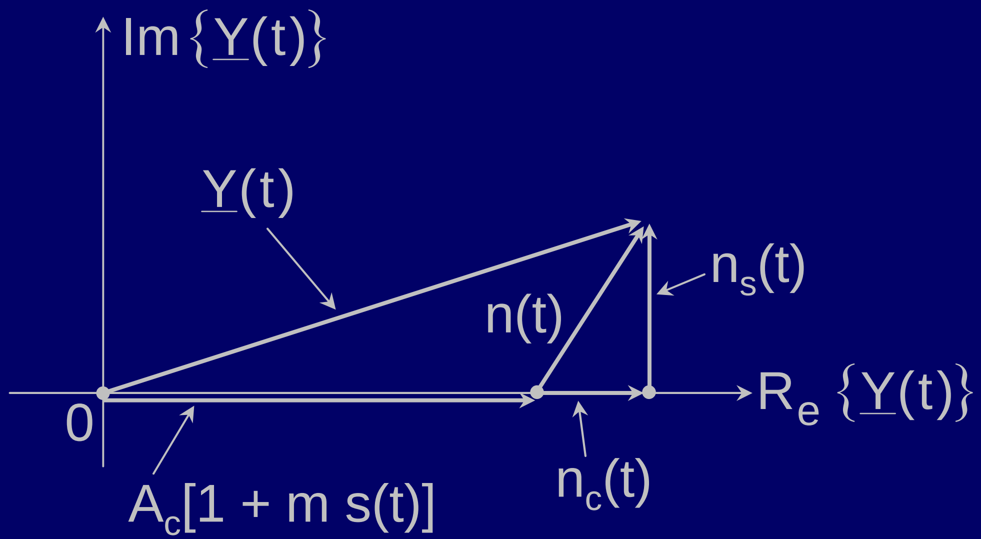
("undermodulation = wasted power)

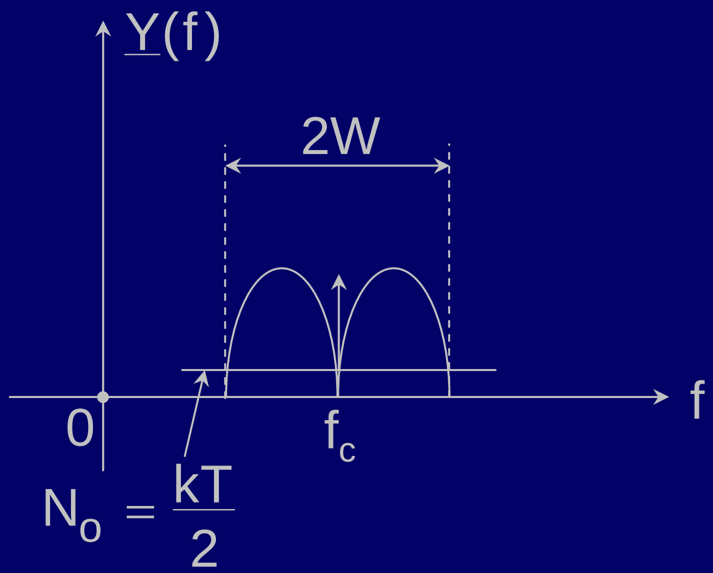
Amplitude Modulation "AM" with Envelope Detector

$$\text{Received} = y(t) = A_c\,[1 + m\ s(t)]\cos\omega_c t + n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$$

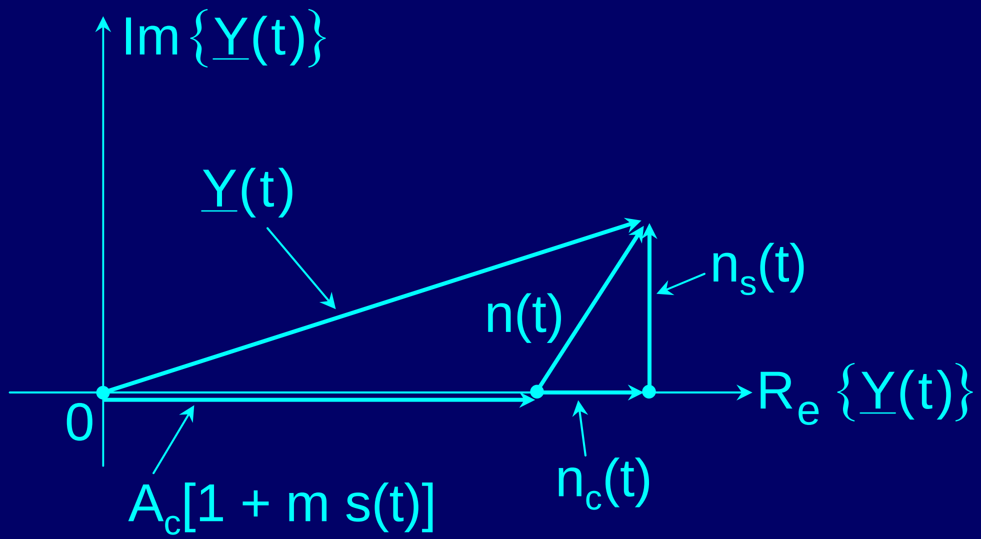
$$= \text{Re}\left\{\underline{Y}(t)e^{j\omega_c t}\right\}$$

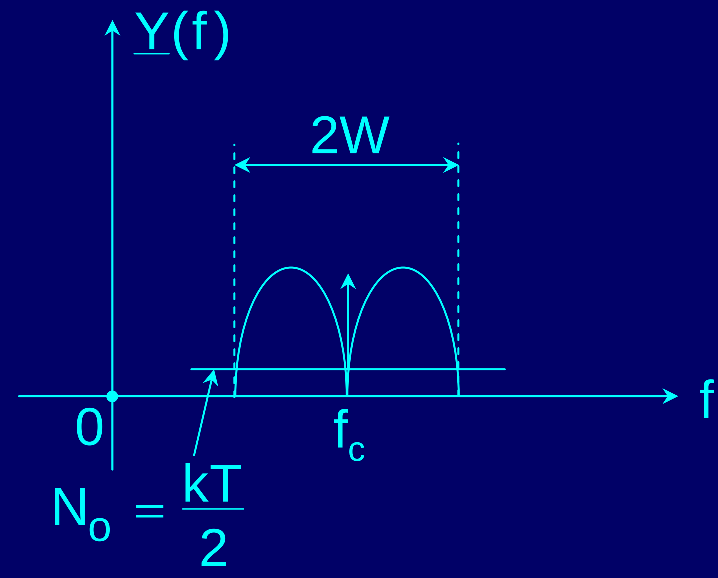
slowly varying





Amplitude Modulation “AM” with Envelope Detector





$$\underbrace{|\underline{Y}(t)|} \cong A_c\,[1+m\;s(t)]+n_c(t)$$

envelope = detected signal + noise

$$\text{Note:}\quad 4WN_o = \left\langle n_c^2\cos^2\omega_c t + n_s^2\sin^2\omega_c t\right\rangle = \left\langle n_c^2\right\rangle$$

$$\frac{S_{out}}{N_{out}} \cong A_c^2 m^2\,\overline{s^2(t)}\Big/\overline{n_c^2(t)} = \frac{A_c^2 m^2\,\overline{s^2(t)}}{4WN_o}$$

Amplitude Modulation "AM" with Envelope Detector

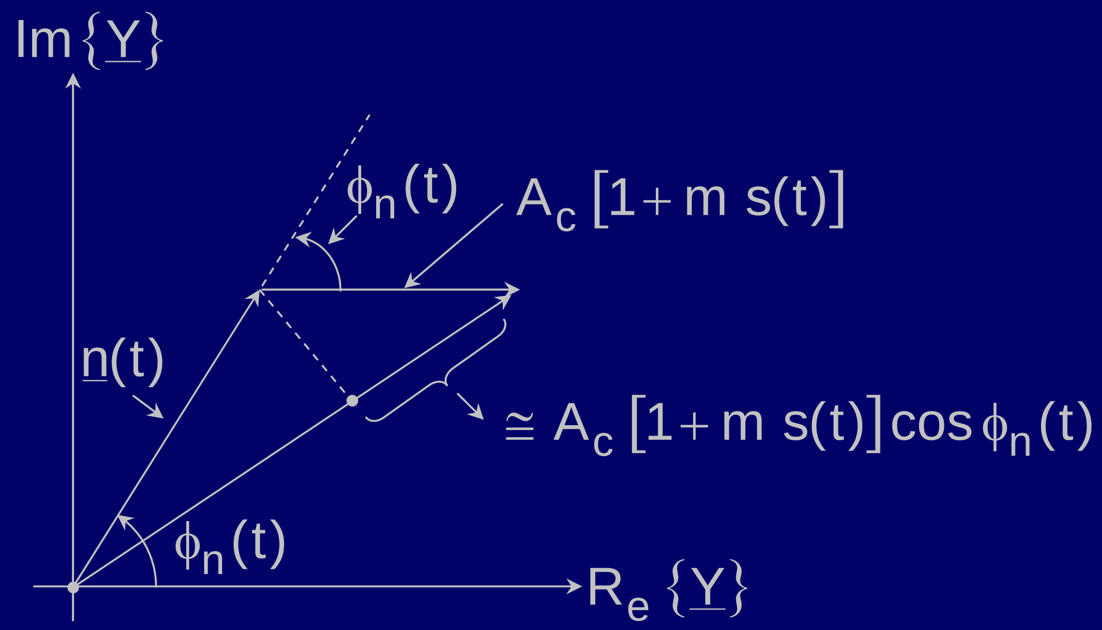
$$\frac{S_{out}}{N_{out}} \cong A_c^2 m^2 \overline{s^2(t)} \Big/ \overline{n_c^2(t)} = \frac{A_c^2 m^2 \overline{s^2(t)}}{4WN_o}$$

$$\frac{S_{in}}{N_{in}} \cong \frac{\left(A_c^2/2\right)\overline{(1+m\ s(t))^2}}{4WN_o} \quad \text{where } S_{in} = \overline{y_{signal}^2(t)}$$

$$\text{Noise figure } F_{AM} \triangleq \frac{S_i/N_i}{S_o/N_o} = \frac{1+m^2\overline{s^2(t)}}{2m^2\overline{s^2}} \gtrsim \frac{1+1/2}{1} = 3/2 \Rightarrow F_{AM} \gtrsim 3/2$$

provided that $A_c >> n_c$ (large S/N limit)

AM Performance (small S/N limit)



$$|\underline{Y}(t)| \cong n(t) + A_c\cos\phi_n(t) + A_c m\, s(t)\underbrace{\cos\phi_n(t)}_{\text{multiplicative noise!}}$$

Want $S_{in}/N_{in} \gtrsim 10$ for fully intelligible AM $\Rightarrow$ "AM threshold"

$\left(\text{i.e. } A_c\,(1+m\,s(t)) \lesssim 3\,n(t)\right)$

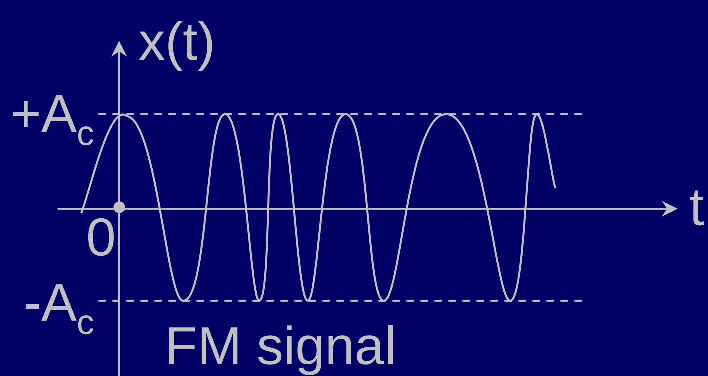
Frequency and Phase Modulation (FM, PM)

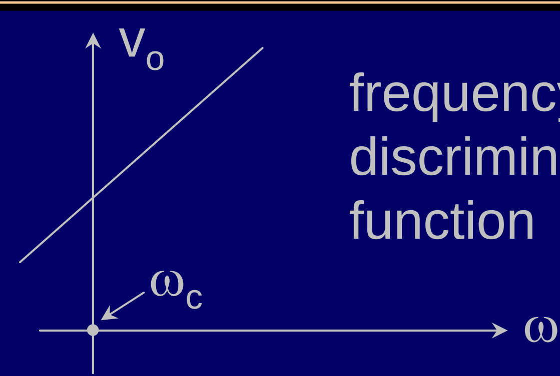
Transmitted: $x(t) = A_c \cos[\omega_c t + \phi(t)]$

Phase Modulation ("PM"): $\phi(t) \triangleq K's(t)$

Frequency Modulation ("FM"):

$$\frac{d\phi}{dt} = \Delta\omega(t) = 2\pi K\ s(t)\ \left(r\ s^{-1}\right) \text{ for } |s(t)| < 1\text{, or } \phi(t) \triangleq \int^t s(\tau)d\tau$$

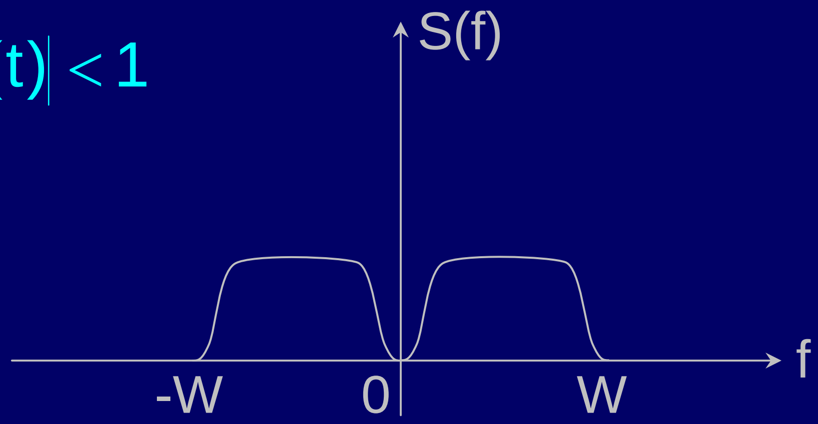




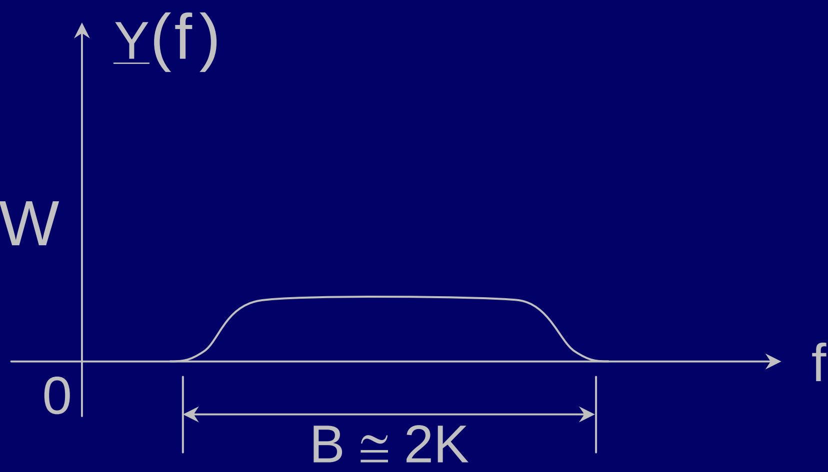
frequency-sensitive discriminator output function

FM Bandwidth Expansion Factor β^*

$$\frac{d\phi}{dt} = \Delta\omega(t) = 2\pi K\ s(t)\ \left[r\ s^{-1}\right] \text{ for } |s(t)| < 1$$



$$\boxed{\beta^* \triangleq K/W}$$

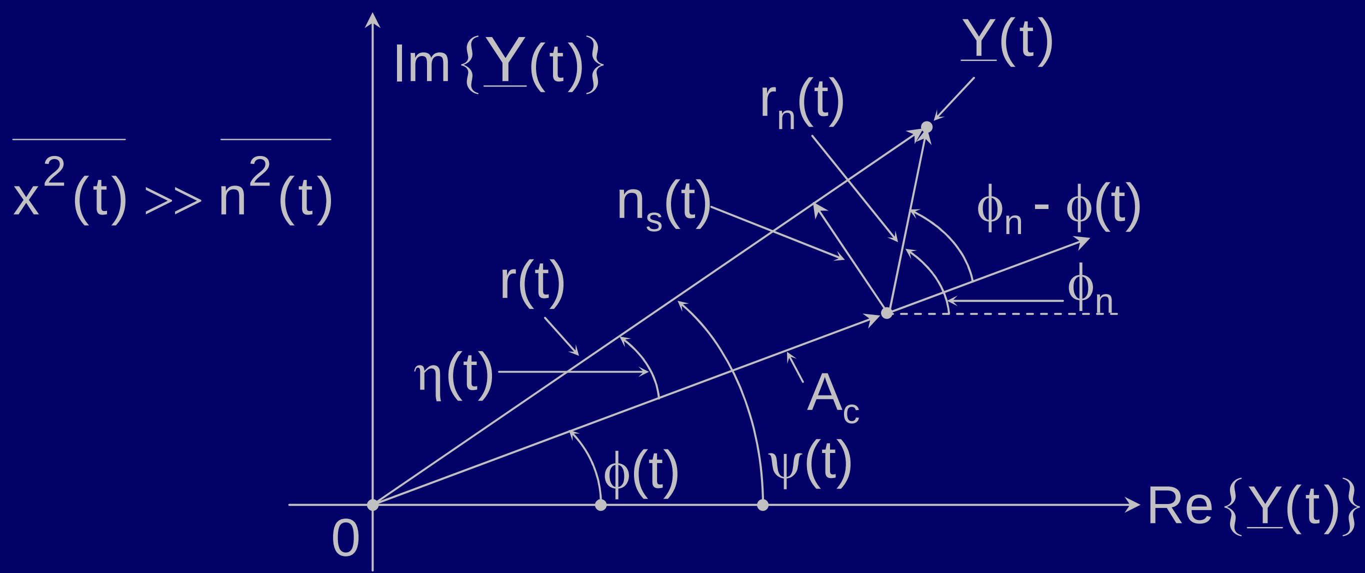


$$B \cong \overbrace{2W}^{\text{intrinsic bandwidth}}\left(1+\beta^*\right)\text{ , so } 2W < B \le 2K + W$$

For PM: $B \cong 2(K'+1)W$ (not proven here) $\left(\text{if } \beta^* >> 1\right)$

Vector Signal Analysis of PM/FM

Received signal $y(t) = x(t) + n(t) = r(t)\cos\left(\omega_c t + \psi(t)\right) = \text{Re}\left\{\underline{Y}(t)e^{j\omega_c t}\right\}$



$$\psi(t) \cong \phi(t) + r_n(t)\sin\left(\phi_n - \phi\right)/A_c \stackrel{\Delta}{=} \phi(t) + n_s(t)/A_c$$

Discriminator output:

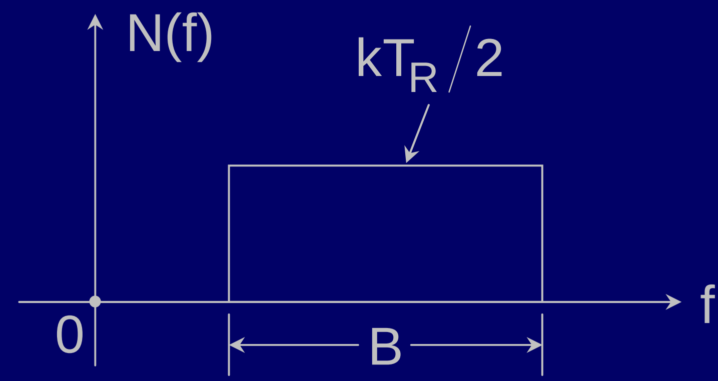
PM: $v(t) = \psi(t) = \phi(t) + \eta(t) = K's(t) + n_s(t)/A_c$

FM: $v(t) = \dot{\psi}(t)/2\pi = Ks(t) + \dot{n}_s(t)/2\pi A_c$ where $\dot{\psi} \stackrel{\Delta}{=} d\psi/dt$

Calculation of PM S_{out}/N_{out}

Recall $n(t) = \underbrace{n_c(t)}_{\text{slowly varying}} \cos\omega_c t - \underbrace{n_s(t)}_{\text{slowly varying}} \sin\omega_c t$

output



$$\overline{n^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)} = 2\underbrace{N_o}_{kT_R/2}B$$

PM: $v(t) = \psi(t) = \phi(t) + \eta(t) = K's(t) + n_s(t)/A_c$

$$\text{Therefore PM: } S_o/N_{out} = \frac{K'^2\,\overline{s^2(t)}}{\left[\underbrace{\overline{n_s^2(t)}}_{2N_oB \cong 4N_oW}/A_c^2\right]} = K'^2\,\overline{s^2(t)} \bullet 2\left[\underbrace{\frac{A_c^2/2}{2N_oW}}_{\text{CNR}}\right]$$

(want large $K'^2\,\overline{s^2(t)}$, but approaches FM)

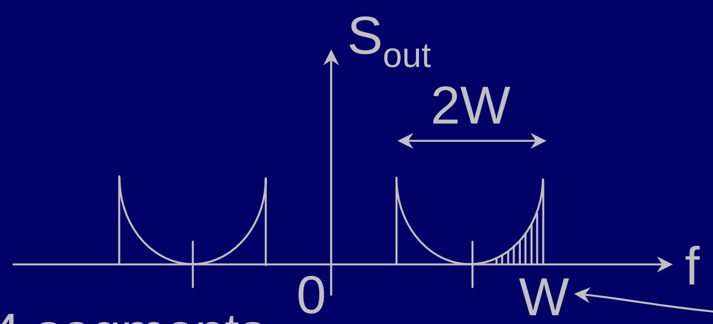
$2N_oB \cong 4N_oW$ for case $B = 2W$ ($K' \ll 1$)

Calculation of FM S_{out}/N_{out}

FM: $v(t) = \dot{\psi}(t)/2\pi = Ks(t) + \dot{n}_s(t)/2\pi A_c$ where $\psi \overset{\Delta}{=} d\psi/dt$

$$
\begin{array}{ccccccc}
n_s(t) \leftrightarrow \underline{N}_s(f) & & \dot{n}_s(t) & \leftrightarrow & j\omega \underline{N}_s(f) \\
\downarrow & \Rightarrow & \downarrow & & \downarrow \\
N_o \overset{\Delta}{=} |\underline{N}_s|^2 & & R_{\dot{n}_s}(\tau) & \leftrightarrow & \omega^2 |\underline{N}_s(f)|^2 = \omega^2 N_o
\end{array}
$$

$$\text{FM}: S_{out}/N_{out} = K^2\overline{s^2(t)} \Big/ \left[\overline{\dot{n}^2(t)}/(2\pi A_c)^2\right]$$



4 segments

$$\left(\frac{1}{2\pi A_c}\right)^2 4\int_0^W (2\pi f)2N_o df = \frac{4N_o}{A_c^2} \bullet \frac{W^3}{3}$$

Calculation of FM S_{out}/N_{out}

$$\text{Therefore } S_{out}/N_{out_{FM}} = K^2\overline{s^2} \bullet \left[\underbrace{\frac{A_c^2}{2}}_{P_c} \bullet 3/2N_oW^3 \right]$$

$$= \frac{3P_c}{2N_oW}\underbrace{\beta^{*2}}_{(K/W)^2}\overline{s^2} = 6[CNR]\beta^{*3}\overline{s^2}$$

where CNR (Carrier-to-Noise Ratio) $= P_c/2N_oB = P_c/2N_o2W\beta^*$

$$\text{"Wide-band FM" (WBFM)} \quad S_{out}/N_{out_{WBFM}} = \underbrace{\left(\frac{S_o}{N_o}\right)}_{P_c\overline{s^2}/2N_oW}{}_{DSBSC} \bullet 3\beta^{*2}$$

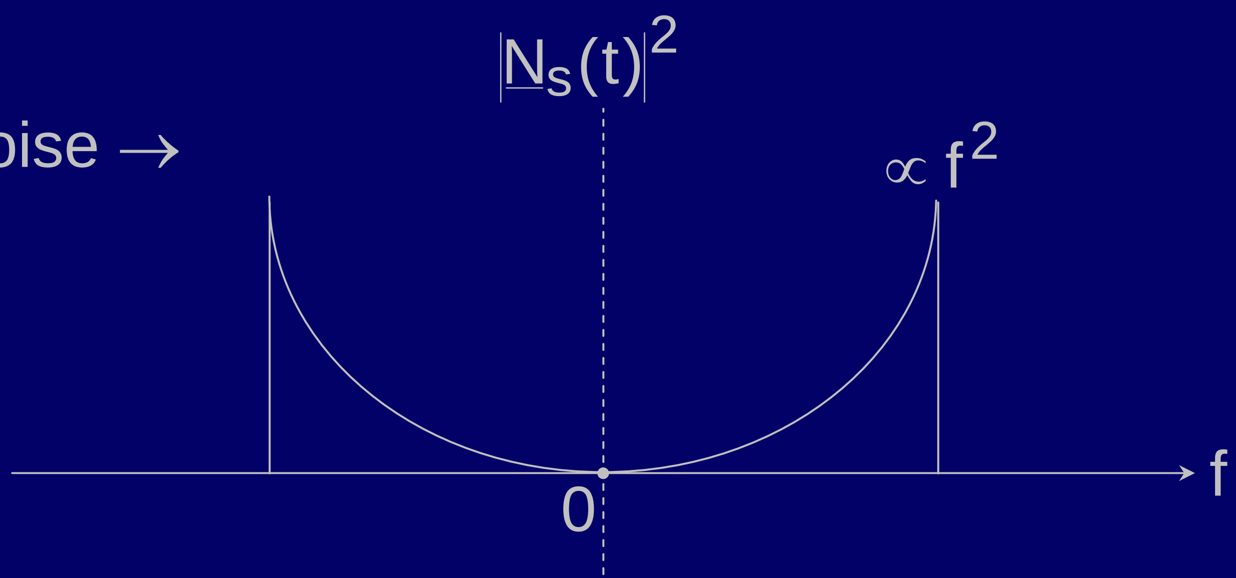
FM advantage for $\beta^* \cong 5$ (FM radio, $2(\beta^*+1)W \cong 200$ kHz);

$3\beta^{*2} \cong 19$ dB!

Calculation of FM S_{out}/N_{out}

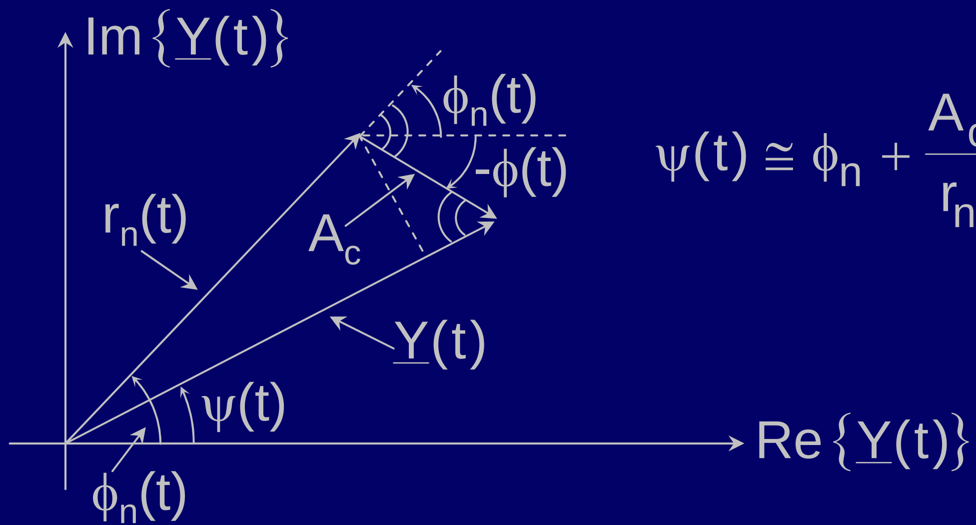
FM pre-emphasis & de-emphasis filters

Since FM noise $\rightarrow$



Pre-emphasis signal $\propto f^2$ pre-transmission and de-emphasize signal + noise at receiver; this can yield ~10 dB improvement (depending…)

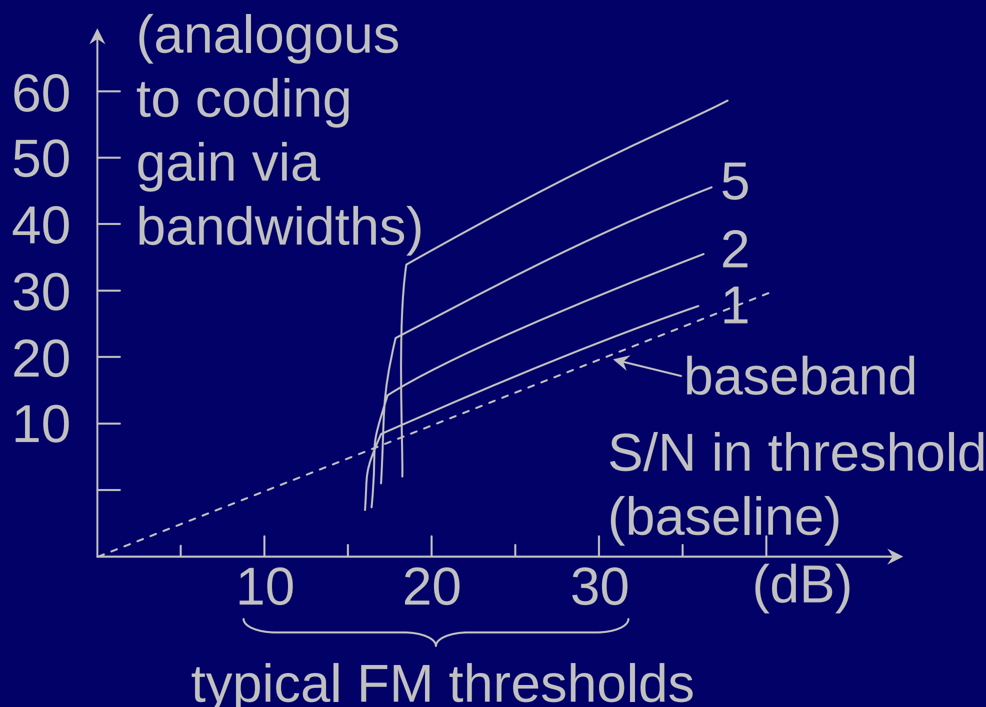
“FM Threshold” – (low SNR limit)

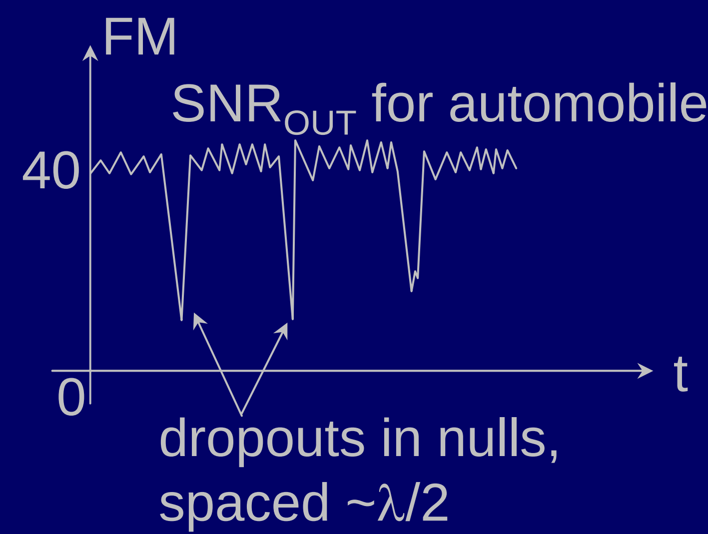


$$\psi(t) \cong \phi_n + \frac{A_c}{r_n}\sin(\phi - \phi_n) \Rightarrow \text{ signal obliteration}$$

ranges over 2π

Therefore must have $A_c >> r_n$ to avoid multiplicative noise





Issues In Choosing Modulation Type

1) Desired output SNR
2) Cost of bandwidth ($, availability) (for communications or storage)
3) Standards imposed on channel, inexpensive equipment
4) Potential for source coding
5) Characteristics (noise, fading), potential for channel coding
6) Cost, power, weight, size, thermal constraints on system

Output SNR Requirements

CD-quality audio:

say 40 dB dynamic range (loudest power/“quiet” power)

+55 dB SNR $\Rightarrow$ 95 dB so 20 $\mathrm{LOG}_{10}\mathrm{L} \cong 95$

Therefore

$\mathrm{L} = 56{,}000$ levels of $\sigma \tilde{\Rightarrow} \; < 32{,}000$ digital levels, $\Rightarrow$ 15-bits

FM-quality audio: say $\beta^* = 5 \Rightarrow$ ~50 dB (~35 dB available above ~15-dB threshold)

Intelligible speech: $\tilde{>}$ 10dB

Video: studio-quality ~40 + dB

Video: home-quality ~20 - 35 + dB

Nominal Bandwidth Requirements

1) Voice: ~3 kHz (6kHz excellent)
2) Music ~15 + kHz
3) Video ~6 MHz (NTSC), 20 MHz (HDTV)
4) Data ~10 – $10^{9\rightarrow 7}$ bits/sec; 10^4 OK often

State-Of-The-Art Source Coding

1) Voice ~1.2, 2.4, 4.8, 9.6 kbps; OK→good

32 – 64 kbps ~ uncompressed

↑ e.g. 8 kHz at $\underbrace{\text{8 bits}}_{\text{~ 58 dB SNR}}$

2) Music 128 – 256 kbps for ~ CD quality, stereo

3) Video 10 kbps jerky, blurred, or little change

56 – 128 kbps ~10 fps, 256^2 pixels (lip-read threshold) and artifacts (moving details)

384 kbps good video conference quality

1.5 Mbps ⇒ "VCR" NTSC TV

6 Mbps ⇒ good NTSC TV

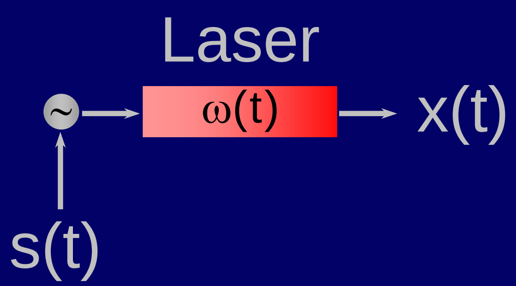
20 Mbps ⇒ HDTV

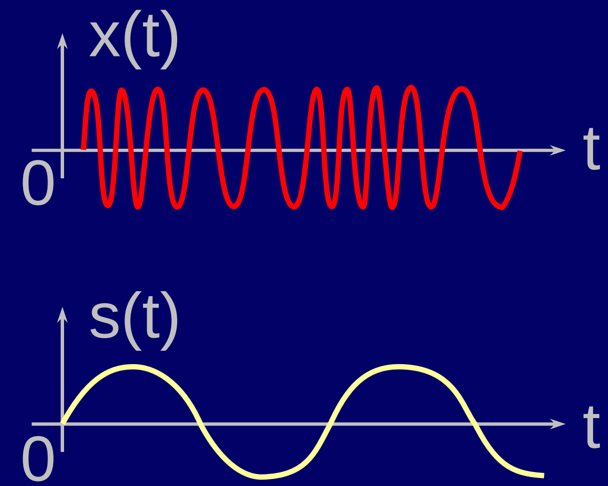
4) Data divide by 2 – 4 for typical miscellaneous data, lossless coding

FM Hybrid Analog Communication System

Laser Example 1

FM Modulation





Here we use the standard definition of CNR for a superheterodyne receiver.

$$S_{OUT}/N_{OUT} = [CNR]\ 6\ \overline{s^2}\ \beta^{*3}\left(\text{where } \beta^* = B/2W\right)$$

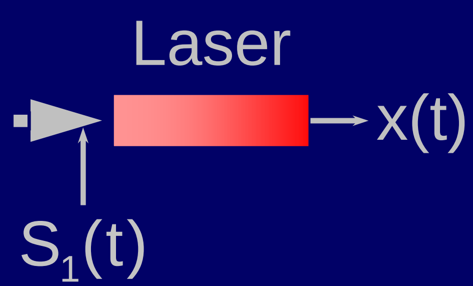
Baseband

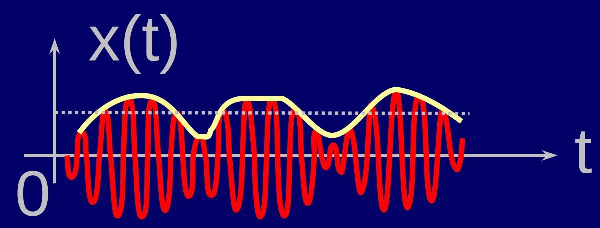
Optical superheterodynes are limited by photon noise that fluctuates with S(t), so the expression here is approximate. Since optical links have great bandwidth, β^* can be very large.

AM Hybrid Analog Communication System

Laser Example 2

AM Modulation





The CNR applies to the unmodulated laser and its detector within the passband of the detector output corresponding to the spectrum of the signal s(t). This CNR must be above the AM threshold of ~10 dB in order for

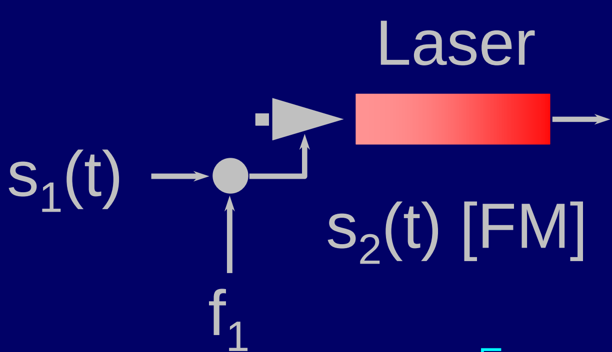
$$S_{OUT} / N_{OUT} = [CNR]\, m^2 \overline{s^2}$$

to apply.

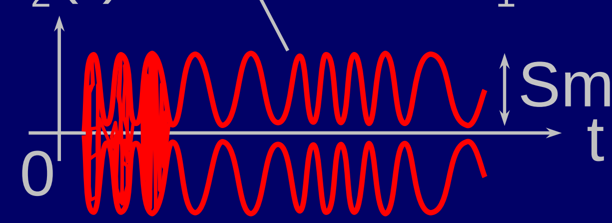
FM/AM Analog Communication System

Laser Example 3

FM/AM Modulation



$s_2(t) \Rightarrow$ AM near f_1 Hz



$f_1(t)$ varies with $s_1(t)$ [FM]

$$S_{OUT}/N_{OUT} \cong [CNR]\left[m^2\overline{s_1^2}\right]_{AM}\left[6\overline{s_2^2}\beta^{*3}\right]_{FM} \quad \text{where } B = 2W\beta^* \text{ for } s_2(t)$$

Assume avalanch photo diode:

$$[CNR]_{APD} = \left(\eta P_s/hf\left[2W\beta^*\right]\right) \Big/ \left(\frac{\langle g^2\rangle}{G^2}\left(1+\frac{P_D}{P_s}\right)+\frac{2kThf}{R_L\eta P_s(eG)^2}\right)$$

P_D = dark current + background power (W). Want $[CNR]\left[m^2\overline{s_1^2}\right]_{AM}$ to be over FM threshold $\cong$ 15 dB; then choose β^* to yield desired S_0/N_O (say 50 dB total)