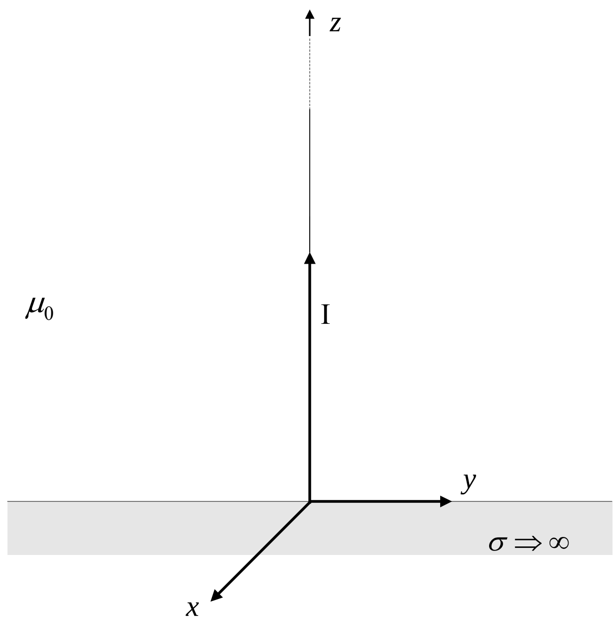
6.641 Electromagnetic Fields, Forces, and Motion
Spring 2009

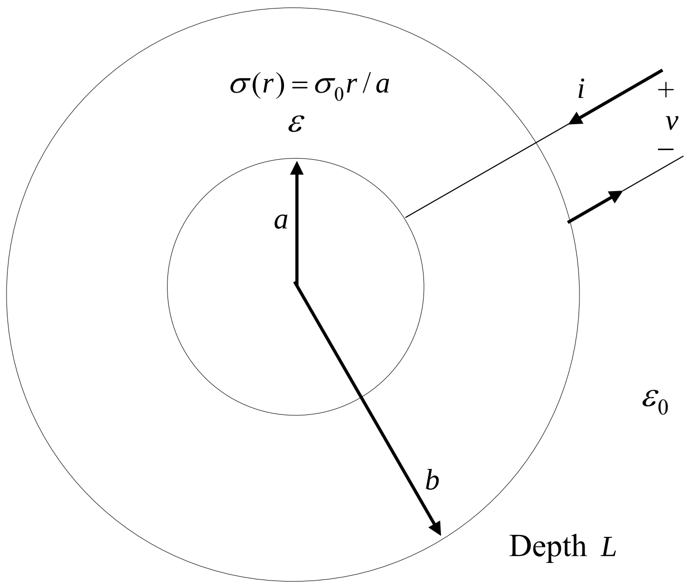
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641, Electromagnetic Fields, Forces, and Motion
Quiz 1
March 18, 2008

The 6.641 Formula Sheet appears at the end of this Quiz. In addition, a 8 1/2" x 11" formula sheet (both sides) that you have prepared is allowed.



1. (30 points) A line current of semi-infinite length stands perpendicularly upon a perfectly conducting ground plane at $z = 0$.

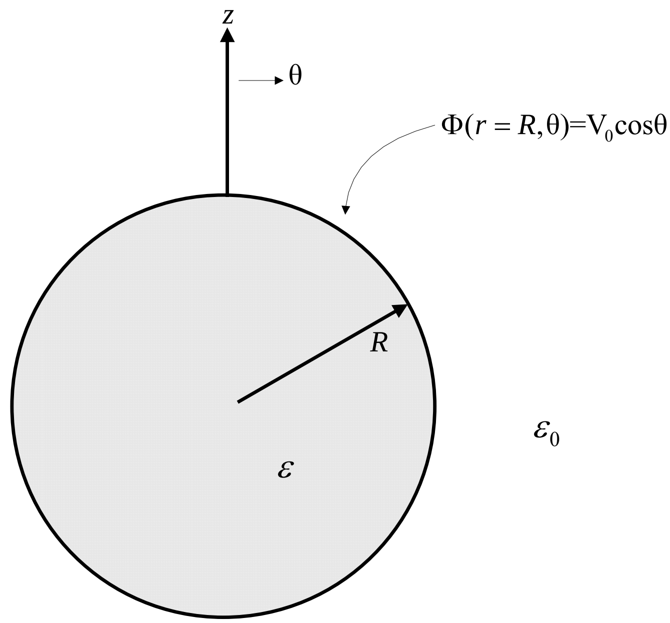
a) Find a suitable image current to find the magnetic field for $z > 0$. Does the direction of the image current surprise you?
b) What is the magnetic field magnitude and direction for $z > 0$?
c) What is the surface current magnitude and direction on the $z = 0$ surface of the conducting plane?



2. (30 points) Concentric cylindrical electrodes with respective radii *a* and *b* and depth *L* are surrounded by free space with permittivity ε_0 and encloses an ohmic material for $a < r < b$ whose conductivity varies with radius as $\sigma(r) = \sigma_0 r / a$ and has constant permittivity ε. A voltage v is applied across the cylindrical electrodes. Neglect end effects at the top and bottom of the cylinder.

   a) What is the electric field for $a < r < b$?
   b) What are the surface charge densities at $r = a$ and $r = b$?
   c) What is the volume charge density for $a < r < b$?
   d) What is the total charge in the system?
   e) What is the resistance between the cylindrical electrodes?

3. (40 points)



A dielectric sphere with permittivity ε and radius R has a potential imposed at $r = R$

$$\Phi(r = R, \theta) = \mathrm{V}_0 \cos\theta$$

Free space with permittivity ε_0 surrounds the sphere for $r > R$.

a) There is no volume charge for $0 < r < R$ and $r > R$ and $\Phi(r = \infty, \theta) = 0$. Laplace's equation for the scalar electric potential in spherical coordinates is:

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial \Phi}{\partial \theta}) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \Phi}{\partial \phi^2} = 0$$

Guess a solution to Laplace's equation of the form $\Phi(r, \theta) = Ar^p \cos\theta$ and find all allowed values of p.

b) Which of your scalar electric potential solutions in part (a) are finite at $r = 0$?

c) Which of your solutions in part (a) have zero potential at $r = \infty$?

d) Using the results of parts (b) and (c) find the scalar electric potential solutions for $0 \le r \le R$ and $r \ge R$ that satisfy the boundary condition $\Phi(r = R, \theta) = \mathrm{V}_0 \cos\theta$.

e) Find the electric field in the regions $0 \le r < R$ and $r > R$.

Hint: $\overline{E} = -\nabla\Phi = -[\frac{\partial \Phi}{\partial r}\overline{i}_r + \frac{1}{r}\frac{\partial \Phi}{\partial \theta}\overline{i}_\theta]$

f) What is the surface charge distribution on the $r = R$ interface?