Problem Set #5   Issued: 3/1/05
Spring Term 2005   Due: 3/17/05

Suggested Reading Assignment: Zahn – 5.1.

Quiz 1 - Wednesday evening, March 9, 2005, 7:30-9:30PM

Covers all material up to and including Problem Set #4 material. The attached 6.641 Formula
Sheet will be supplied with Quiz 1. One 8½"x11" sheet of notes (both sides) that you prepare
can be brought with you into the exam.

Problem 5.1

A point charge q with mass m and velocity $\bar{v}$ moves in a vacuum through a magnetic field $\bar{H}$.
Newton's law for this charge is:

$$m\frac{d\vec{v}}{dt} = q\vec{v} \times \mu_o \vec{H}$$
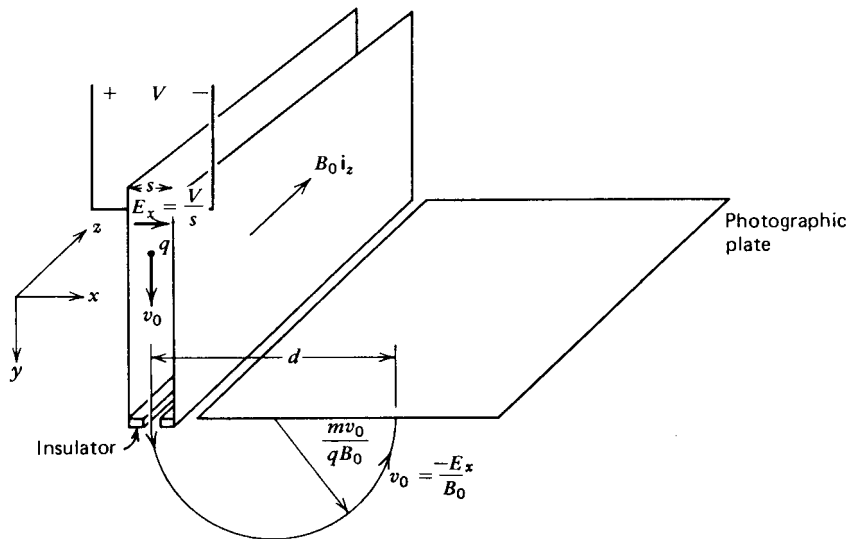
A uniform magnetic field in the z direction is imposed

$$\vec{H} = H_o \vec{i}_z$$

Solve Newton's law for the three velocity components v_x, v_y , and v_z for initial conditions

$$\vec{v}(t=0) = v_{xo}\vec{i}_x + v_{yo}\vec{i}_y + v_{zo}\vec{i}_z$$

Note that the velocity magnitude is constant so that a circular motion results in the x-y plane.
What is the radius of the circle?

a)

Diagrams from: *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, Robert E. Krieger
Publishing Company, 1987. Used with permission.

The mass spectrograph uses the circular motion to determine the masses of ions and to measure
the relative proportions of isotopes. Charges enter between parallel plate electrodes with a y
directed velocity distribution. Gravitational effects are negligible compared to the
electric/magnetic forces since the ions have so little mass.

i. To pick out those charges with a particular magnitude of velocity, perpendicular electric and magnetic fields are imposed so that the net force on a charge is

$$\bar{f} = q(\bar{E} + \bar{v} \times \mu_o \bar{H})$$

For charges to pass through the narrow slit at the end of the channel, they must not be deflected by the fields which require this force to be zero. For a selected velocity $v_y = v_o$, what is the required applied electric field and thus the necessary voltage V for a given magnetic field $H = H_o \bar{i}_z$ so that an ion will make it through this slit undeflected? The plate electrodes have a spacing, s, as shown in the figure.

ii. Note that the charge circular path diameter d depends on the ion mass, and so can be used to detect different isotopes that have the same number of protons but a different number of neutrons. The isotopes thus have the same charge but different masses. Typically V=-100 volts across a s=1 cm gap with a magnetic field of μ_o H=1 tesla. The mass of a proton and neutron are each about m=1.67 x 10^{-27} kg. Consider the three isotopes of magnesium $12Mg^{24}$, $12Mg^{25}$, and $12Mg^{26}$, each deficient of one electron. At what positions d will each isotope hit the photographic plate?

b)
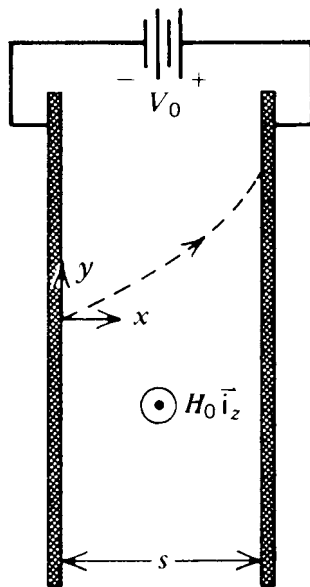
A magnetron is essentially a parallel plate capacitor stressed by constant voltage V_o where electrons of charge -e are emitted at $x=0$, $y=0$ with zero initial velocity. A transverse magnetic field $H_o \bar{i}_z$ is applied. Neglect the electric and magnetic fields due to the electrons in comparison to the applied field.

i. What is the velocity and displacement of an electron, injected with zero initial velocity at $t=0$?

ii. What value of magnetic field will just prevent the electrons from reaching the other electrode? This is the cut-off magnetic field.

Problem 5.2
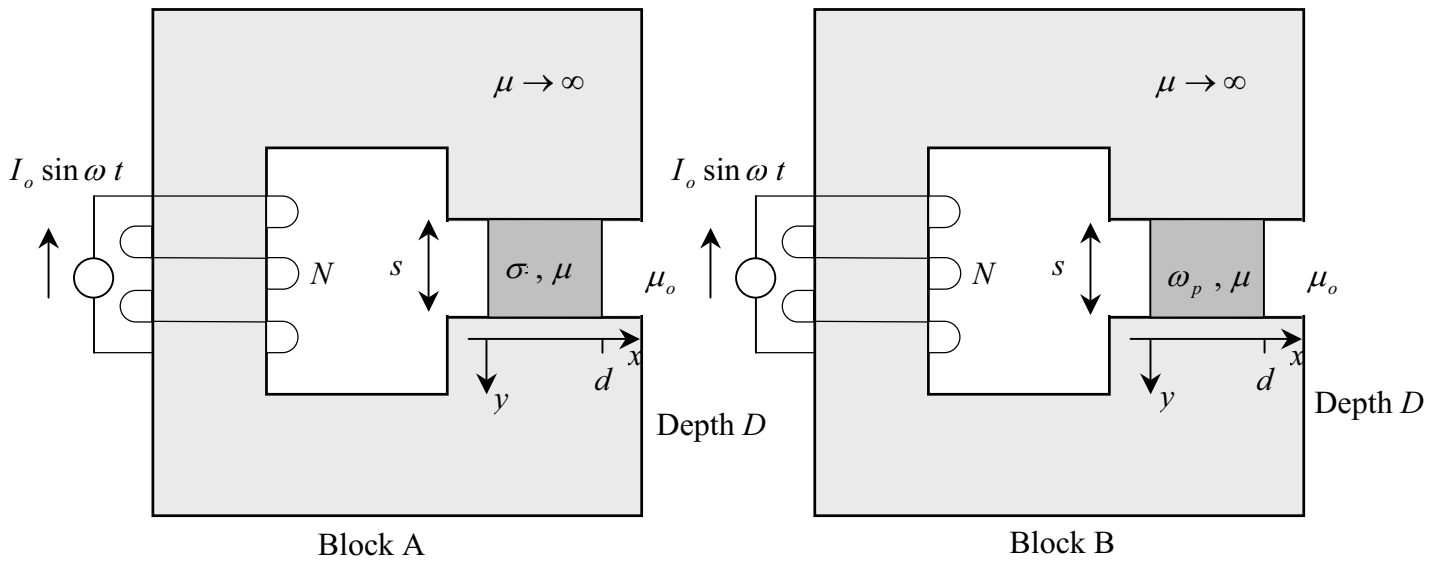
Block A   Block B

Diagrams from: *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, Robert E. Krieger Publishing Company, 1987. Used with permission.

Two different blocks of material are placed within identical magnetic circuits with exciting currents $I_o \sin \omega t$. Block A is described by a constant ohmic conductivity while Block B is a superconductor described by a plasma frequency ω_p. Both blocks have constant magnetic permeability μ and permittivity ε. The conduction constitutive laws for these blocks are:

$$\bar{J} = \sigma \bar{E} \qquad\qquad \text{Block A (Ohmic)}$$

$$\frac{\partial \bar{J}}{\partial t} = \omega_p{}^2 \varepsilon \bar{E} \qquad\qquad \text{Block B (Superconductor)}$$

a) Use Ampere's integral law to find the magnetic field H_y in the free space region within the air-gap just to the left and right of the blocks.
   Hint: There are $\pm z$ directed current flows in the blocks but the net z directed current is zero.

b) Use these conduction constitutive laws with the MQS system form of Ampere's law to obtain
$$\nabla \times \bar{H} = \bar{J} = \sigma \bar{E} \qquad\qquad \text{Block A}$$
$$\nabla \times \frac{\partial \bar{H}}{\partial t} = \frac{\partial \bar{J}}{\partial t} = \omega_p{}^2 \varepsilon \bar{E} \qquad\qquad \text{Block B}$$

c) Take the curl of the results in (b) and use Faraday's law to obtain
$$\nabla^2 \bar{H} = \sigma \mu \frac{\partial \bar{H}}{\partial t} \qquad\qquad \text{Block A}$$
$$\nabla^2 \bar{H} - \frac{\omega_p{}^2}{c^2} \bar{H} = 0 \qquad\qquad \text{Block B}$$

What is c?

d) Take the magnetic field in each of the blocks to be of the form

$$\bar{H} = H_y(x)\sin\omega t\, \bar{i}_y$$

Solve (c) for the magnetic field distribution in each of the blocks using the solutions in (a) as boundary conditions at $x=0$ and $x=d$. Assume that there are no surface currents at $x=0$ and $x=d$. Plot $|H_y(x)|$ versus x.

e) What is the current distribution, $\bar{J}$, in each of the blocks? Plot $|\bar{J}|$ versus x.

Suggestion: For the plots, you may wish to normalize

$$H_y \text{ by } \frac{NI_0}{s}, \ x \text{ by } d, \ J \text{ by } \frac{NI_0}{s}\sqrt{\frac{\omega\mu\sigma}{2}} \text{ for Block A}$$

$$J \text{ by } \frac{\omega_p NI_0}{cs} \text{ for Block B}$$

Problem 5.3

A sheet of surface charge at $x=0$ is of infinite extent in the y and z directions. It has surface charge distribution:

$$\sigma_s = \sigma_o \cos ay$$

a) What are the potential and electric field distributions for $x<0$ and $x>0$?
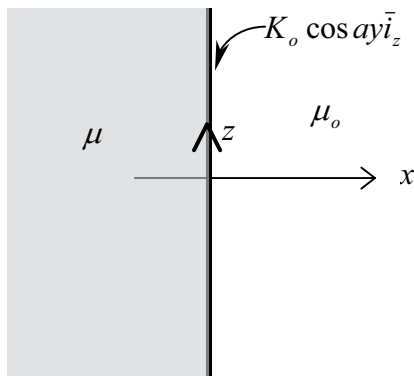   What is the equation of the electric field line

$$\frac{dy}{dx} = \frac{E_y}{E_x}$$

which passes through the point $(x=0, y=y_0)$?
Using your favorite computing drawing program, plot the equipotential and field lines in the x-y plane.

Problem 5.4

A current sheet $K_o \cos(ay)\bar{i}_z$ is placed at $x=0$ at the interface between a magnetic material with permeability μ and free space. Because there are no volume currents for $x \neq 0$, a scalar magnetic potential ψ can be defined such that

$$\bar{H} = -\nabla\psi .$$

Diagrams from: *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, Robert E. Krieger Publishing Company, 1987. Used with permission.

a) What is the general form of solution for ψ for all space?
b) What boundary conditions must be satisfied? Solve for the unknown constants in part (a).
c) What is the magnetic field $\bar{H}$ everywhere?