

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 7
Fall Term 2006

6.630 Electromagnetics

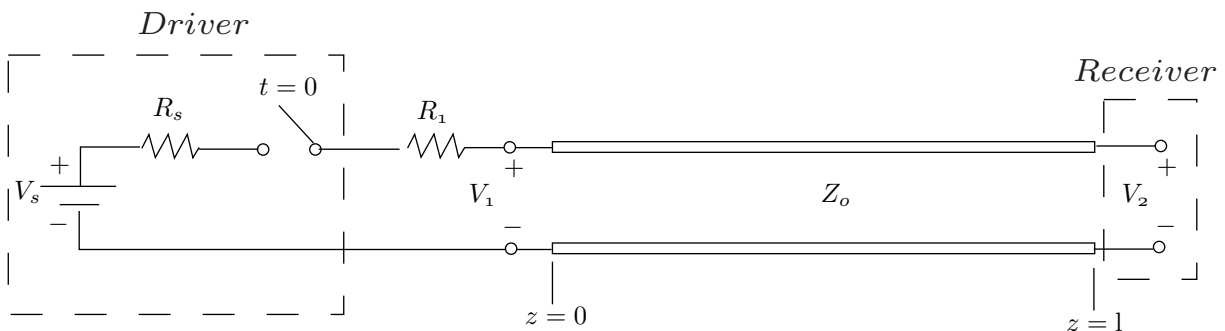
Issued: Week 8
Due: Week 9

Reading assignment: Section 2.3, 2.5; J. A. Kong, “*Electromagnetic Wave Theory*”.

Problem P7.1

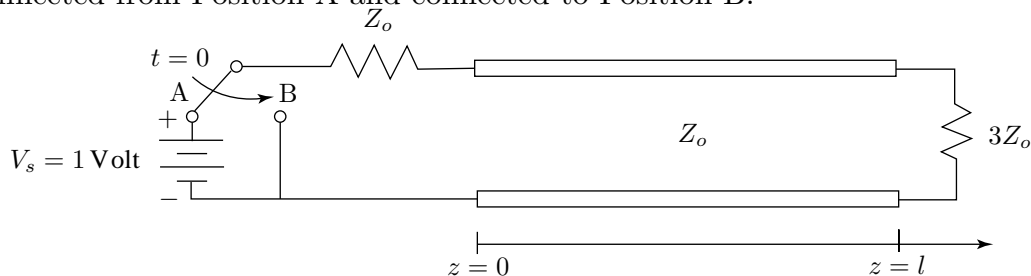
Digital systems on printed circuit boards often incorporate source terminations instead of load terminations to minimize the dissipated power and to reduce the magnitude of propagating voltage waveforms. Consider the digital system in the figure. The driver is modeled by a voltage source $V_s = 1$ V and a source resistance $R_s = 25 \Omega$. The transmission line has a characteristic impedance $Z_0 = 100 \Omega$ and a propagation velocity $v = c/2 = 1.5 \cdot 10^8$ m/s due to the geometry and dielectric constant of the the printed circuit board. Digital receivers often have very low input capacitance so that the load can be modeled as an open circuit in the frequency range of interest. The source termination is implemented by inserting a series resistance $R_1 = 75 \Omega$ so that the total input resistance matches the characteristic impedance of the transmission line.

- The length of the transmission line is measured to be 30 cm. Calculate the round trip propagation time for a signal transient.
- A low to high transition is modeled by a switch that closes at $t = 0$. If you measure the voltage at V_1 , you might conclude that the quality of the signal is poor. Plot the voltage at V_1 versus time assuming zero volts initially on the line.
- Now assume that the signal quality measurement is made at the digital receiver. Plot the voltage V_2 versus time assuming zero volts initially on the line.



Problem P7.2

Consider a transmission line circuit shown in the following figure. At $t = 0$, the switch is disconnected from Position A and connected to Position B.



- Find the reflection coefficient at the load (at $z = l$).

(b) Make labelled sketches of the total voltage $V(z)$ on the line, $0 < z < l$, at

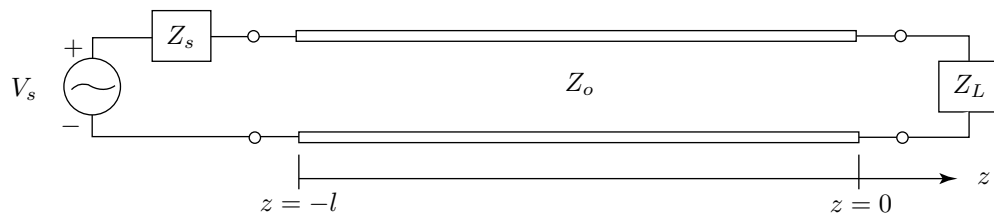
- (i) $t < 0$
- (ii) $t = l/2v$
- (iii) $t = 2l/v$

where v represents the velocity of propagation on the transmission line.

Problem P7.3

Consider a TEM transmission line as shown in the following figure. The characteristic impedance of the transmission line is Z_0 and its length is $l = 2\lambda$, where λ is the wavelength in the line. The load impedance is Z_L . The current on the line is given by

$$I(z) = I_o \cos kz.$$



- (a) What is V_+ and what is V_- in terms of Z_0 and I_o ?
- (b) What is the reflection coefficient at the load Γ_L ?
- (c) Show that the load impedance $Z_L = 0$.
- (d) The real voltage in space and time is defined as $V(z, t) = \text{Re} \{V(z)e^{j\omega t}\}$. Let I_o and Z_0 be real, write down the expression and sketch the voltage on the line at $\omega t = \pi/2$.
- (e) Let the voltage of the source be $V_s = I_o Z_0$, what is the source impedance Z_s in terms of Z_0 ?