

6.551J/HST714J Problem Set 2

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6.551J/HST714J Acoustics of Speech and Hearing

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How much time did you spend on this problem set? \_\_\_\_\_

**Problem 1: Real and Imaginary numbers and complex conjugates**

a) In lecture 1, we defined a complex conjugate  $\underline{P}^*$  such that:

$$|\underline{P}^*| = |\underline{P}|, \text{ and } \angle \underline{P}^* = -\angle \underline{P}.$$

Use your knowledge of complex numbers to prove that given:  $\underline{P} = a + jb$ , then  $\underline{P}^* = a - jb$ .

b) A common error in the use of complex numbers is to equate:

$$\text{Real}\left\{\frac{1}{\underline{P}}\right\} \text{ with } \frac{1}{\text{Real}\{\underline{P}\}}$$

Prove that these two quantities are not equal. (Hint: assume  $\underline{P} = a + jb$ .)

**Problem 2. Summing pressure waves of the same frequency**

An example of the utility of complex amplitudes is to use them to sum cosine waves of identical frequency but varying amplitude and phase. Consider the series:

$$p_1(t) = 2 \cos(\omega t),$$

$$p_2(t) = 1 \cos(\omega t + \pi),$$

$$p_3(t) = 1 \cos(\omega t + \pi/2), \text{ and}$$

$$p_4(t) = 2 \cos(\omega t - \pi/2).$$

Define.

$$p_{SUM}(t) = p_1(t) + p_2(t) + p_3(t) + p_4(t).$$

What are the amplitude and phase of  $p_{SUM}(t)$ ?

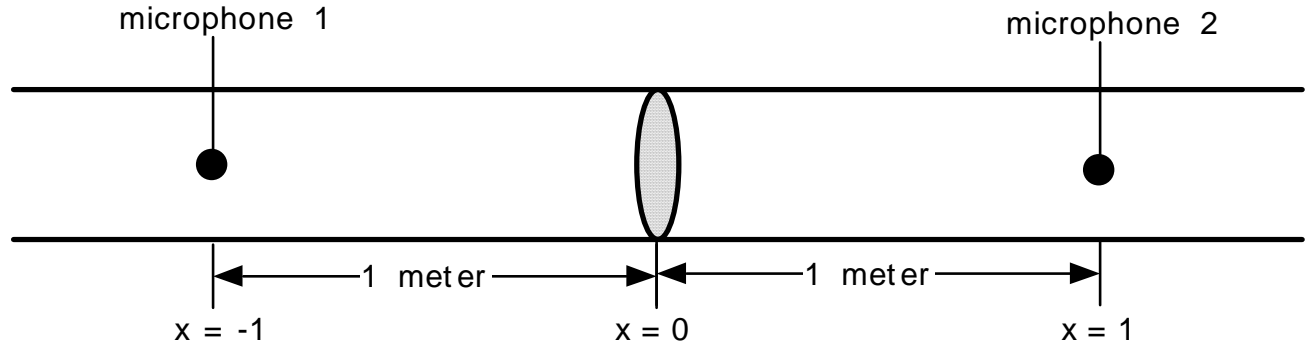
One way of doing the problem is to:

define 4 complex amplitudes  $\underline{P}_1 \dots \underline{P}_4$  with appropriate amplitude and phase,  
compute the real and imaginary parts of the 4 complex amplitudes,  
compute the sum of the four complex numbers, and  
compute the magnitude and angle of the sum.

A second way of doing the problem is to graphically do a 'vector' sum of the four complex numbers and measure the resultant magnitude and angle.

A third way is to use a computer to prepare digital representations of the 4 waves, which you then sum.

Use one of these or your own to compute your solution.

**Problem 3: Plane Waves in a Symmetric Tube**

Consider the infinitely long cylindrical tube above with a flat solid plate positioned orthogonally to the long axis of the tube at position  $x=0$ . The plate has an area equivalent to the cross-sectional area of the cylinder. Two microphones are symmetrically placed around the plate, so that each is centered in the tube 1 meter from the plate, but microphone 1 is on the left and microphone 2 is on right. Suppose the plate has been moving with a sinusoidal velocity for some time and that at time  $t=0$  the plate is in sine phase, such that  $v_x(t)=A \cos(2\pi 1000t - \pi/2)$ , where  $A=1$  mm/s.

5.a Sketch a plot that describes the velocity of the piston during the period  $t=0$  through 5 ms.

The motion of the piston produces an acoustic disturbance that propagates down the tube, and you can assume that the particle velocity in air near the piston has a magnitude equal to the piston's velocity.

5.b First concentrate on the sound wave that propagates to the right ( $x \geq 0$ ). Sketch a plot that describes the sound induced  $x$  component of the particle velocity  $v_x(t,x)$  inside the tube as a function of position for the range  $0 \leq x \leq 1$  meters, when  $t=0, 0.25$  and  $0.5$  ms.

5.c Sketch a plot that describes the time wave form of the sound pressure measured by microphone 2 during the period  $t=0$  through 5 ms. You can assume that the specific acoustic impedance of the air in the tube equals the characteristic impedance of the medium.

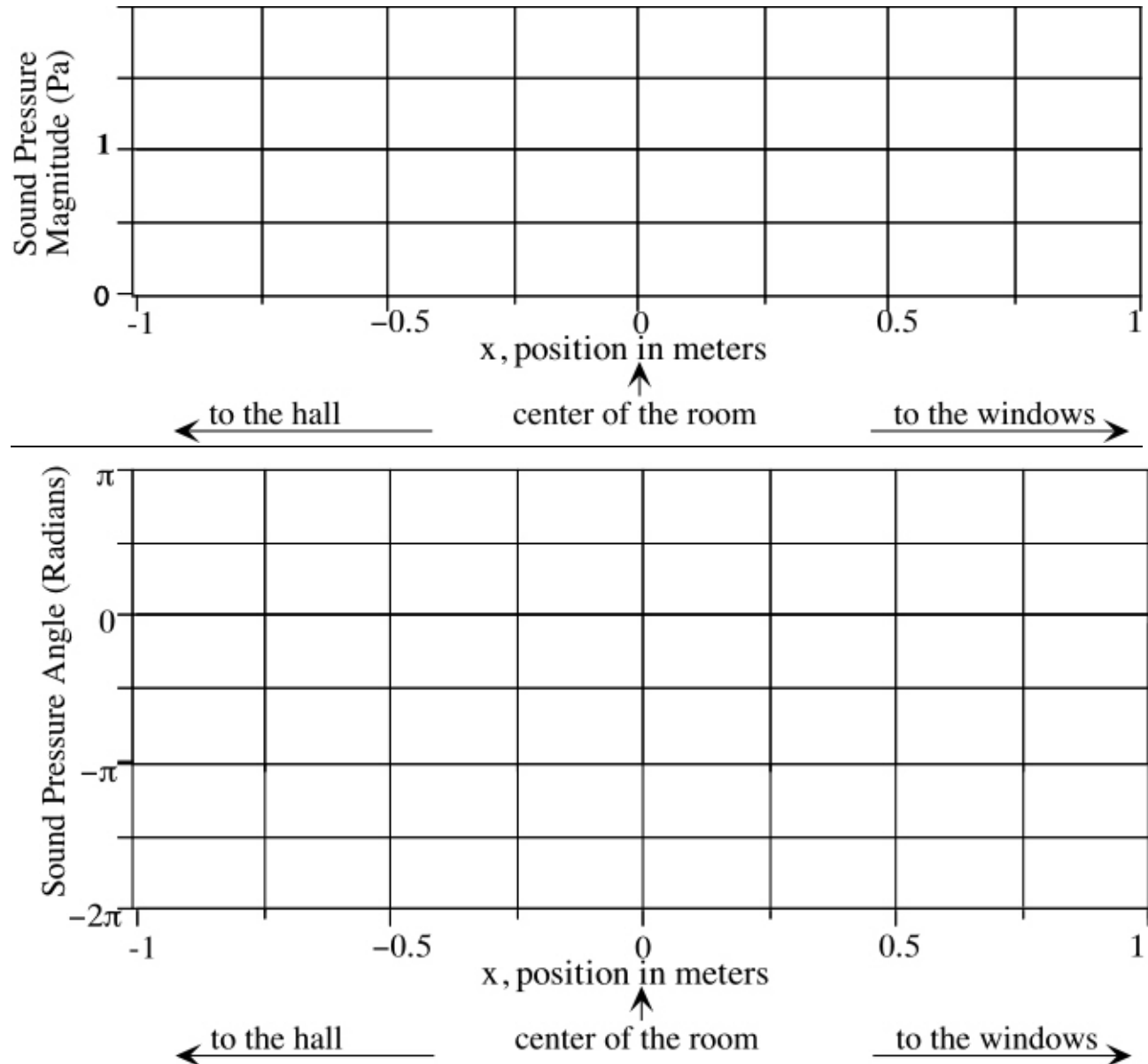
5.d Now concentrate on the sound wave that propagates to the left  $x \leq 0$ . Sketch a plot that describes the sound induced  $x$  component of the particle velocity inside the tube as a function of position for the range  $-1 \leq x \leq 0$  meters, when  $t=0, 0.25$  and  $0.5$  ms.

HINT: Be mindful of the different phase of the source in (b) & (d). While the source for (b) is  $v_x(t)=A \cos(2\pi 1000t - \pi/2)$ , the *effective* source driving the leftward waves in (d) is  $v_x(t)=-A \cos(2\pi 1000t - \pi/2)$ . You'll also need to think clearly about waves traveling in the negative  $x$  direction.

**Problem 4: Bi-directional Plane Waves**

Consider a plane wave propagating across the front of a Room much like we talked about in Lecture 2. Assume the forward going plane wave is the result of a distant sinusoidal source of 340 Hz that is somewhere to the left of the room. The wave then propagates from left to right across the front of the room as observed by you. We can define the variation in sound pressure in time and space by:

$p(x,t) = \text{Real}\{P e^{-jkx} e^{j\omega t}\}$ , where  $P = 1$  pascal,  $\omega = 2\pi 340$ ,  $k = \omega/c$ , (assume  $c = 340$  m/s),  $x=0$  is at a point in the middle of the room, and that  $x$  increases from left to right.



- What is the magnitude and angle of the complex amplitude of the forward traveling wave at  $x=0$ ?
- Sketch, on the above graphs, how the magnitude and phase angle of the sound pressure  $P e^{-jkx}$  varies as a function of  $x$ . (Remember that  $\sin(0) = \sin(2\pi) = \sin(4\pi) \dots$ ).

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Now assume that there is an impedance ‘zero’ located orthogonal to the direction of propagation of the plane wall located at position  $x=0$ . This zero produces a reflected ‘backward’ traveling wave of complex amplitude  $-Pe^{jkx}$ . Note that negative sign in front of the pressure in the backward traveling wave constrains the ‘backward’ wave pressure to be of equal amplitude but opposite sign to the pressure in the forward traveling wave at  $x=0$ .

- c. Relate the equal and opposite pressures in the two waves at  $x=0$  and the impedance zero at  $x=0$ ?
- d. Sketch, on the above graph, how the magnitude and phase angle of the sound pressure  $p^-(x,t)$  in the backward traveling wave (described by the complex amplitude above) vary as a function of  $x$ .
- e. Now suppose both waves are traveling at the same time. You can determine the magnitude and angle of the summed waves at each location by adding the vectors that describe the magnitude and angle of the two pressure waves at each location. How does the sum differ from its parts? Are their regular variations in magnitude of either the parts or the sum? What about regular variations in angle?

Another way to think of the variation of magnitude and phase of the sound pressure with  $x$  in this case is to start with the complex amplitude that describes the sum of the two waves:

$$\underline{P}(x) = Pe^{-jkx} - Pe^{jkx}$$

This equation is readily reducible to a single sine function in  $x$ . How?

Hint: Factor out the magnitude of the two waves and then use Euler’s equations to expand each of the exponentials into a combination of cosine and sine functions. A little algebra should then yield:

$$\underline{P}(x) = -j2P \sin(kx)$$

- f. How does the predicted sine function in space compare with your graphical description of the magnitude and angles of the traveling waves above? Specifically:  
How does the magnitude of the summed pressures vary in space?  
If there are nulls in this spatial distribution, where do they occur?

### Problem 5. Spherical waves

A microphone is placed in a sound field produced by a simple source of radius  $a$  that generates uniform spherical waves described by a sinusoidal spherical traveling wave

$$p(r,t) = \frac{A}{r} \cos(2\pi 4000(t - r/c)).$$

The sound field is bounded by anechoic walls that absorb all sound energy and generate no reflections, so we need only consider outward traveling waves.

- a). Use exponential notation to describe the dependence of the sound pressure and particle velocity on  $r$ .
- b). When the microphone is located at  $r_{80} = 2$  m, the sound pressure at the microphone is 80 dB SPL and when located at  $r_{40}$  the sound pressure is 40 dB SPL. Determine the value of  $r_{40}$ .

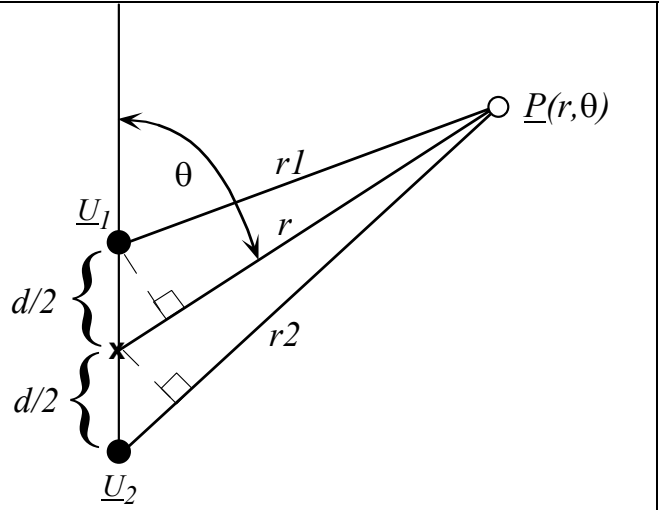
**Problem 6: Two spherical sources**

Two loud speakers that produce continuous pure tones of 500 Hz are placed in space as in the figure to the right with  $d=0.34$  meters. The source strength of each speaker is

$$\underline{U}_1 = \underline{U}_2 = U_S = 0.1e^{j0}.$$

We are interested in determining the sound pressure produced by the speakers at positions 30 meters from the center point of the speaker pair, i.e.  $r=30\text{m}$ , and at varied angles  $\theta$ .

Please assume that the propagation velocity of sound is 340 m/s.



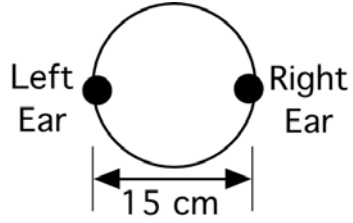
- What is the sound pressure (magnitude and angle) produced by speaker 1 alone at position  $r = 30\text{m}$ ,  $\theta = 90$  degrees?
- What is the sound pressure (magnitude and angle) produced by speaker 2 alone at position  $r = 30\text{m}$ ,  $\theta = 90$  degrees?
- What is the sound pressure (magnitude and angle) produced by simultaneous activation of both speakers at position  $r = 30\text{m}$ ,  $\theta = 90$  degrees?
- What is the sound pressure (magnitude and angle) produced by speaker 1 alone at position  $r = 30\text{m}$ ,  $\theta = 0$  degrees?
- What is the sound pressure (magnitude and angle) produced by speaker 2 alone at position  $r = 30\text{m}$ ,  $\theta = 0$  degrees?
- What is the sound pressure (magnitude and angle) produced by simultaneous activation of both speakers at position  $r = 30\text{m}$ ,  $\theta = 0$  degrees?
- The sound pressure at  $r = 30\text{m}$ ,  $\theta = 0$  degrees depends strongly on the separation of the two speakers. Given that physical constraints do not permit  $d$  to be less than 0.1 meter, is there some value of  $d$  where the sound pressure at  $r = 30\text{m}$ ,  $\theta = 0$  degrees approximates that at  $r = 30\text{m}$ ,  $\theta = 90$  degrees? What is  $d$  under those conditions?

**Problem 7: Sound-Diffraction and Localization**

Listeners use differences in interaural (the difference between the two ears) phase and intensity in order to localize the sources of tones. At frequencies below about 1000 Hz, a reasonable model of the human head in localization is a set of microphones separated by a distance of about 1.5 times the anatomical interaural distance (figure 7b). In a paragraph or two, discuss why such a model yields reasonable predictions of the interaural differences in tone phase and intensity at frequencies below 1000 Hz. Include some statement about whether such a model would yield reasonable interaural differences in phase and intensity at higher frequencies ( $1000 < f < 8000$  Hz).

**Figure 7:**

a. A spherical model of the human head



b. A low-frequency model of Interaural Phase & Intensity

