LECTURE 7

Last time:

- Huffman codes

Lecture outline

- Elias codes
- Slepian-Wolf
- Compression: pulling it together

Reading: Scts. 5.8-5.9, 14.4 through the end of 14.4.4.1.

Huffman codes

Huffman codes are optimal

The algorithm we have builds from the least likely elements onwards

Do we need to view the whole codeword to decode?

Would such a code be more or less robust to slight errors in distribution?

In general, how important is it to know the actual distribution?

Still want to make frequent elements short and infrequent one longer.

Elias codes

Encoder has a sliding structure

The length of the codeword is variable

Take a binary string 011010111010

we can consider it as a real 0.011010111010

Suppose the probability assignment is 0.7 for 0 and 0.3 for 1

We can create a description of the source-word as follows:

divide the interval [0,1)

as new bits are read, subdivide the the specified interval further in proportion to the probabilities

Mapping the sourceword to the codeword

For the sourceword, after $n-1$ source bits, an interval $[a_{n-1}, b_{n-1})$ has been specified

Bit n specifies a new interval $[a_n, b_n)$ such that

If bit n is 0, then $a_n = a_{n-1}$ and $b_n = a_{n-1} + p(b_{n-1} - a_{n-1})$

If bit n is 1, then $a_n = a_{n-1} + p(b_{n-1} - a_{n-1})$ and $b_n = b_{n-1}$

The codeword divides interval in halves

We do not necessarily need to receive the whole codeword to start decoding

Length of the codeword

We make the codeword long enough to determine in which part of the sourceword interval we are

That is to say that the codeword interval must be *inside* the sourceword interval

$$
\begin{aligned}
2^{-n} &\leq P_{\underline{X}^n}\left(\underline{x}^n\right) \\
-n &\leq \log\left(P_{\underline{X}^n}\left(\underline{x}^n\right)\right) \\
n &\geq \log\left(\frac{1}{P_{\underline{X}^n}\left(\underline{x}^n\right)}\right)
\end{aligned}
$$

which is satisfied by taking

$$n = \left\lceil \log\left(\frac{1}{P_{\underline{X}^n}(\underline{x}^n)}\right)\right\rceil + 1$$

Length of the codeword

The average length is then:

$$\sum_{\underline{x}^n \in \mathcal{X}^n} P_{\underline{X}^n}(\underline{x}^n) \left(\left\lceil \log \left(\frac{1}{P_{\underline{X}^n}(\underline{x}^n)} \right) \right\rceil + 1 \right)$$

$$\leq H(\underline{X}^n) + 2$$

This is very close to the limit given by the AEP

Hoewever, for short sequences, this may not be particularly good