Recitation-2 Solutions

Section III Example 1

i) Z = 40.
ii) 1/8
iii) 3/5

Section III Example 2

i) 9/634

ii) Most of the probability mass is concentrated in a single assignment viz. set all nodes equal 0.

The most likely assignment with X5 taking the value '1' is is $X_5 = 1; X_{rest} = 0$. This is already very less probable as compared to the MAP assignment (to be precise, its probability is 9/625 times that of MAP). Other assignments will have even lower probabilities. Thus, just by observation, we can say that the most likely assignment for node 5 is state 0.

iii) This question was supposed to be a prelude to the elimination algorithm. Now that we have learnt about it, the problem should be relatively easy.

If we use the brute force method of summing over all variables, we will need to do $|X|^9$ computations, or roughly $2^9 = 512$ computations. However, using the elimination algorithm, we can bring down the number of operations significantly. Since this is a 3x3 grid graph, any elimination ordering will create a clique of size at least (3+1) = 4 in the reconstituted graph. One such ordering is: 1,2,3, 4,6, 7,8,9.

Thus, the best complexity we can achieve is $|X|^4$ or 16 computations. (However, note that these summations over 4 variables will have to be done multiple times, so the actual number of operations will be higher. Still, it will be *much* smaller than 512. A loose upper bound to the number of operations can be as follows: $N|X|^4 = 9 * 16 = 144$ operations.)

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