Problem Set 10

The Short-Time Fourier Transform and Modulated Filter Banks

**Issued:** Tuesday November 22, 2005.   **Due:** Tuesday December 6, 2005.

Reading: Chapter 10, sections 10.3 and 10.5. This problem set also includes a problem on multirate filter banks. There is no coverage of this in the text, but the lecture slides and your notes from lecture should be sufficient.

Note: This problem set is *not* due on November 29, as was first advertised. Although it is now due on the same date as Project II, it's our hope that in addition to the reduced length of this problem set, the later submission date will allow for added flexibility in completing both.

Problem 10.1

The system in Figure 10.1-1 uses a modulated filter bank for spectral analysis. (For further illustration, Figure 10.1-2 shows how the H's relate.) The impulse response of the prototype filter $h_0[n]$ is sketched in Figure 10.1-3.
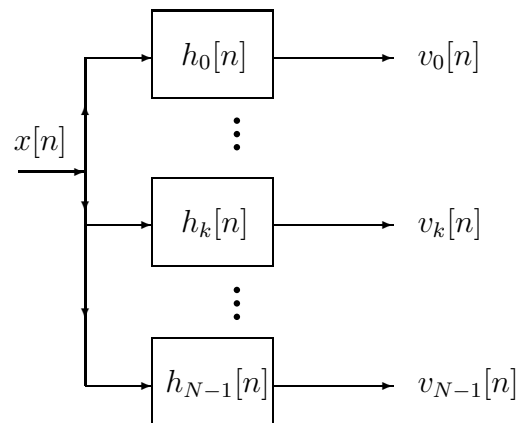
$$h_k[n] = e^{j\omega_k n}h_0[n], \qquad \omega_k = \frac{2\pi k}{N}, \qquad \text{where } k = 0, 1, ..., N-1$$

$$h_0[n] = \text{lowpass prototype filter}, \qquad H_k(z) = H_0(e^{-j\frac{2\pi k}{N}}z)$$
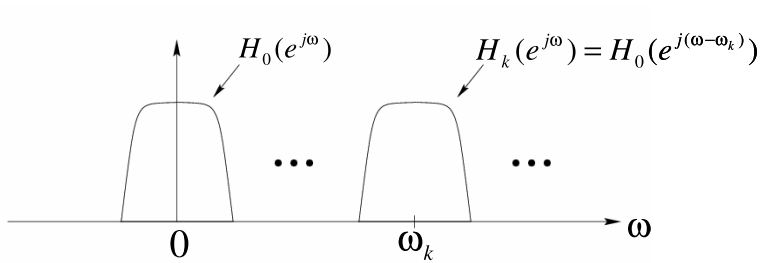
Figure 10.1-1

$$H_0(e^{j\omega}) \qquad H_k(e^{j\omega}) = H_0(e^{j(\omega - \omega_k)})$$

Figure 10.1-2

$$h_0[n] = \begin{cases} 0.9^n & 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases}$$

where, $M < N$
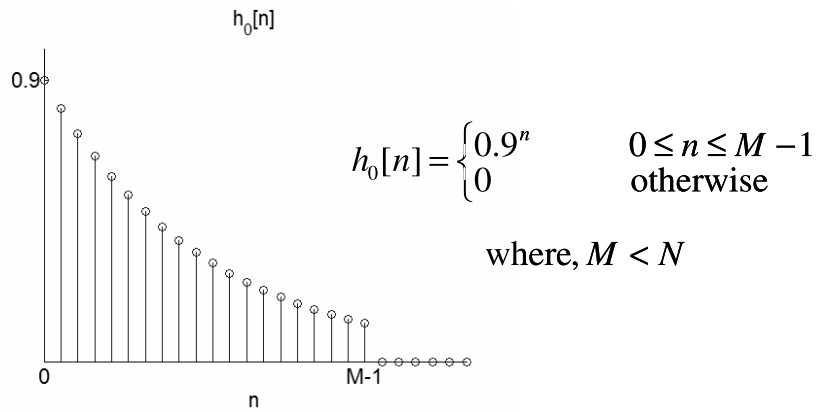
Figure 10.1-3

An alternative system for spectral analysis is shown in Figure 10.1-4. Determine $w[n]$ so that $G[k] = v_k[0]$, for $k = 0, 1, ..., N-1$.
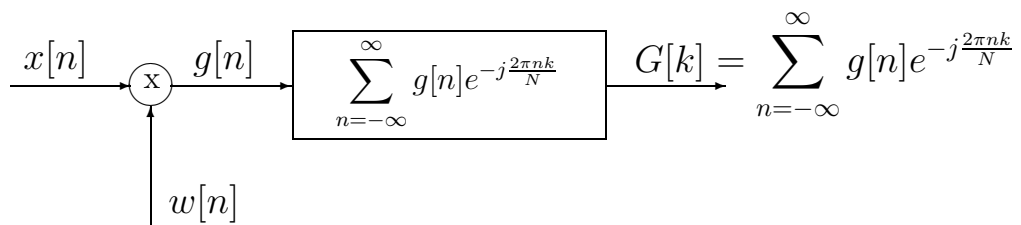

$$x[n] \quad \otimes \quad g[n] \quad \boxed{\sum_{n=-\infty}^{\infty} g[n] e^{-j\frac{2\pi nk}{N}}} \quad G[k] = \sum_{n=-\infty}^{\infty} g[n] e^{-j\frac{2\pi nk}{N}}$$

$$w[n]$$

Figure 10.1-4

Problem 10.2

We are interested in obtaining 256 equally-spaced samples of the z-transform of $x_w[n]$. $x_w[n]$ is a windowed version of an arbitrary sequence $x[n]$ where $x_w[n] = x[n]w[n]$ and $w[n] = 1$, $0 \leq n \leq 255$ and $w[n] = 0$ otherwise. The z-transform of $x_w[n]$ is defined as

$$X_w(z) = \sum_{n=0}^{255} x[n]z^{-n}.$$

The samples $X_w[k]$ that we would like to compute are

$$X_w[k] = X_w(z)\big|_{z=0.9e^{j\frac{2\pi}{256}k}} \qquad k = 0, 1, \ldots, 255.$$

We would like to process the signal $x[n]$ with a modulated filter bank, as indicated in Figure 10.2-1
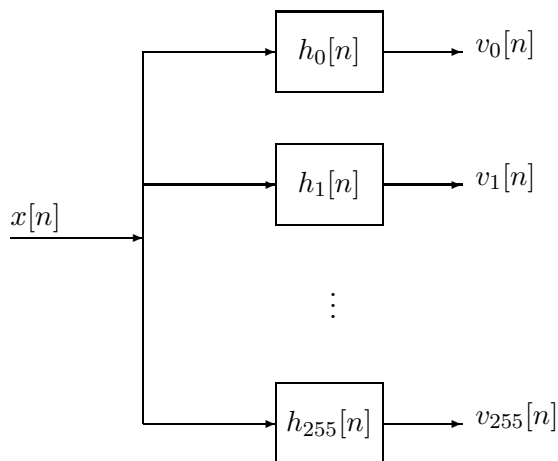

Figure 10.2-1

Each filter in the filter bank has an impulse response that is related to the prototype *causal* lowpass filter $h_0[n]$ as follows:

$$h_k[n] = h_0[n]e^{-j\omega_k n} \qquad k = 1, 2, \ldots, 255.$$

Each output of the filter bank is sampled once, at time $n = N_k$, to obtain $X_w[k]$, i.e.

$$X_w[k] = v_k[N_k].$$

Determine $h_0[n]$, ω_k and N_k so that

$$X_w[k] = v_k[N_k] = X_w(z)\big|_{z=0.9e^{j\frac{2\pi}{256}k}} \qquad k = 0, 1, \ldots, 255.$$

Problem 10.3

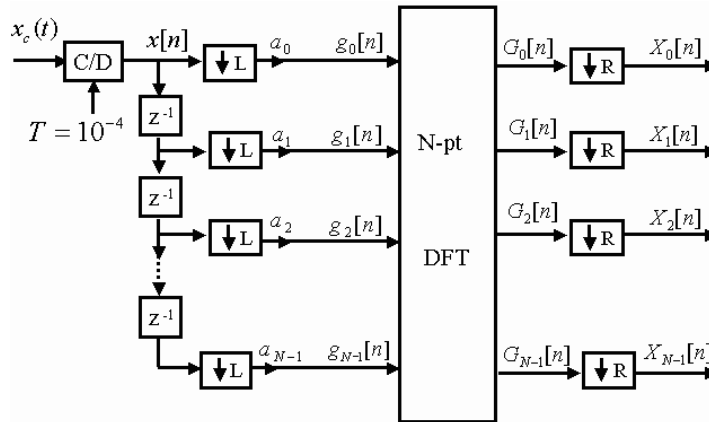In Figure 10.3-1, we show a system for spectral analysis of a signal $x_c(t)$,

Figure 10.3-1

where

$$G_k[n] = \sum_{l=0}^{N-1} g_l[n] e^{-j\frac{2\pi}{N}lk},$$

$$N = 512, \quad \text{and} \quad LR = 256.$$

(a) For the most general choice of the multiplier coefficient a_l, determine the choice for L and R which will result in the smallest number of multiplies per second.

In Figure 10.3-2, we show another system for spectral analysis of a signal $x_c(t)$,
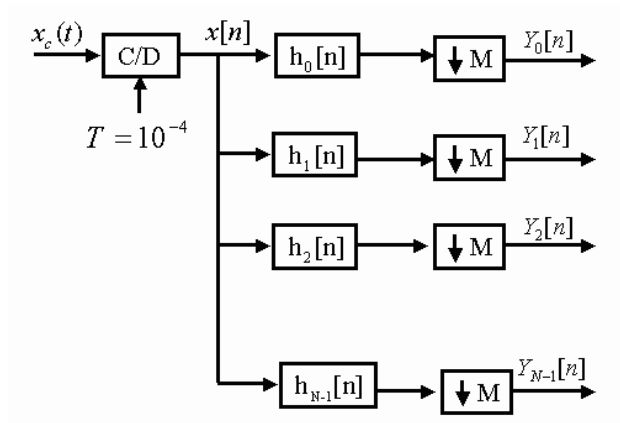

Figure 10.3-2

where

$$h[n] = \begin{cases} (0.93)^n & 0 \le n \le 255 \\ \\ 0 & \text{otherwise} \end{cases},$$

$$h_k[n] = h[n]e^{-j\omega_k n}, \qquad k = 0, 1, \cdots, N - 1, \text{ and}$$

$$N = 512.$$

(b) Listed below are **two** possible choices for M, **four** possible choices for ω_k, and **six** possible choices for the coefficients a_l. From this set identify all combinations for which $Y_k[n] = X_k[n]$, i.e., for which both systems will provide the same spectral analysis. There may be more than one.

M: (a) 256   (b) 512

ω_k :   (a) $\frac{2\pi k}{256}$ (b) $\frac{2\pi k}{512}$ (c) $\frac{-2\pi k}{256}$ (d) $\frac{-2\pi k}{512}$

a_l: (a) $(0.93)^l$ l=0, 1, $\cdots$, 255,   zero otherwise

(b) $(0.93)^{-l}$ l=0, 1, $\cdots$, 511

(c) $(0.93)^l$ l=0, 1, $\cdots$, 511

(d) $(0.93)^{-l}$ l=0, 1, $\cdots$, 255,   zero otherwise

(e) $(0.93)^l$ l=256, 257, $\cdots$, 511,   zero otherwise

(f) $(0.93)^{-l}$ l=256, 257, $\cdots$, 511,   zero otherwise

Problem 10.4

OSB Problem 10.40, (a) - (d)