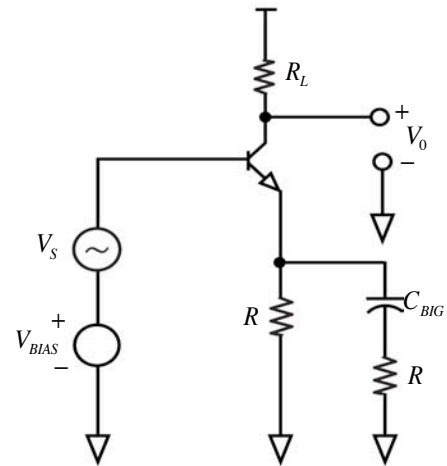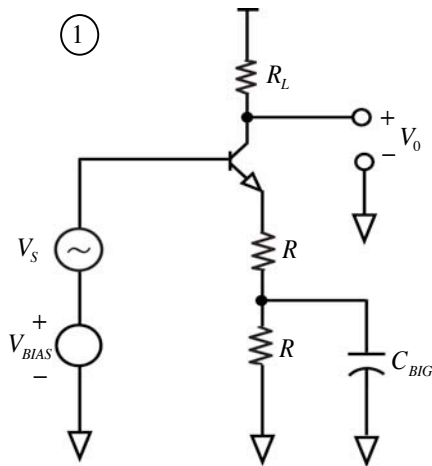# 6.301 Solid State Circuits

Recitation 7: Emitter Degeneration, and More on Multistage Amps
Prof. Joel L. Dawson

We're going to look at emitter degeneration in detail today. The purpose is in part to review, and in part to help pull together a few of the concepts that we've dealt with in the class up to this point. To kick things off, then…

CLASS EXERCISE
Draw mid-band small-signal equivalents for the following two circuits:

(Workspace)

6.301 Solid State Circuits
Recitation 7: Emitter Degeneration, and More on Multistage Amps
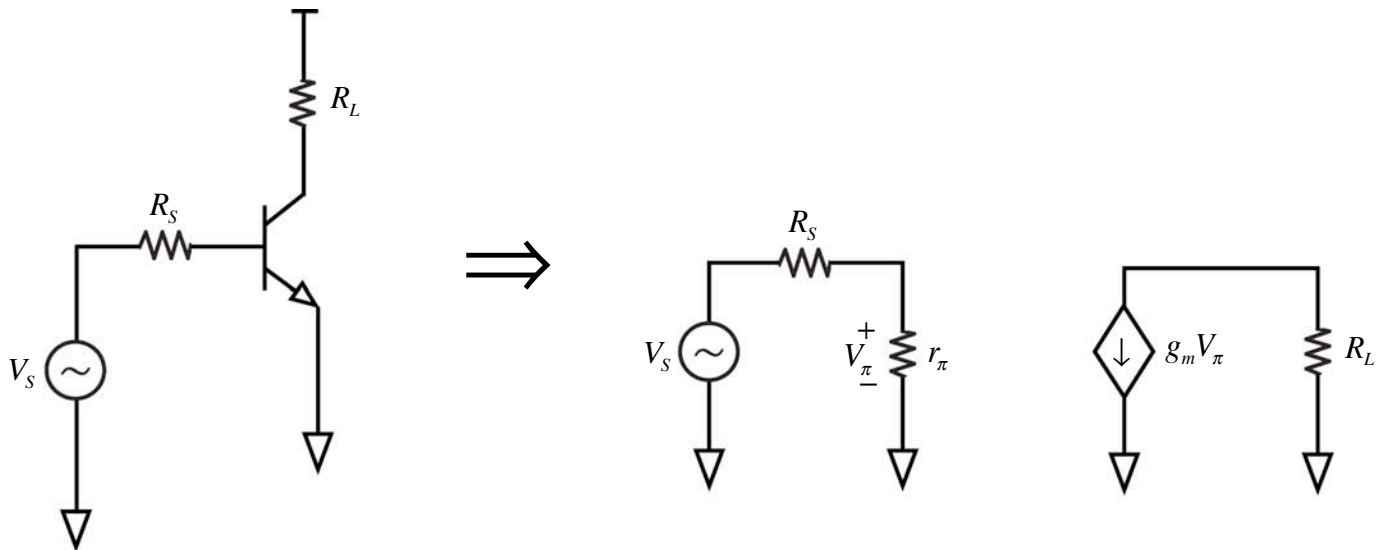Prof. Joel L. Dawson

It is true that emitter degeneration complicates the circuit analysis compared to the common emitter amplifier. Why do we bother? Because of the things that we gain. In exchange for:

- Reduced gain
- Reduced voltage headroom

We gain

- Increasing independence of stage gain from characteristics of the transistor $\left(a_v \approx \dfrac{R_L}{R_E} \right)$

- Increased input impedance
- Increased bandwidth (lower OCTC sum associated w/stage)

So how do we deal with the analysis? One way is to write out KCL and KVL and crunch. But analysis always has a better chance of coming out right when it is guided by a firm "conceptual compass." Look at the common emitter amplifier:
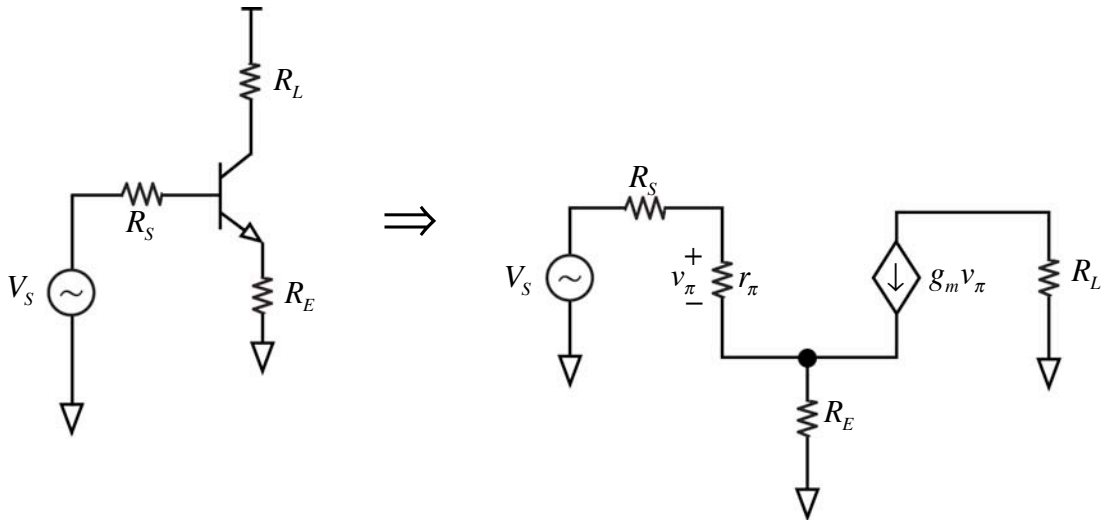
And compare to the emitter degenerated case:

What we would like to do is somehow transform the network above to one that looks like the simple common-emitter stage. What does that mean, exactly?

Well, R_S and R_L mean the same thing in both diagrams. Otherwise, an "equivalent" network must have the same input impedance, gain, and output impedance. Let's look at input impedance first.
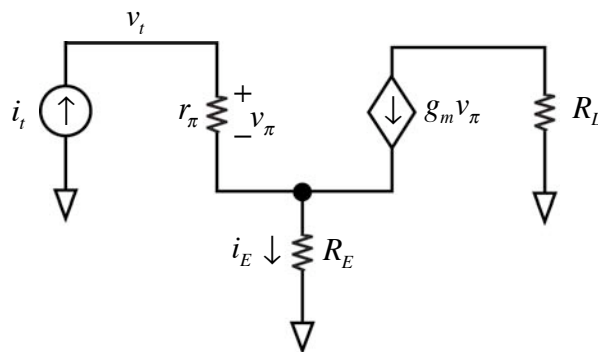
$$v_t = v_\pi + (i_t + g_m v_\pi)R_E$$
$$v_\pi = i_t r_\pi$$
$$v_t = i_t r_\pi + (i_t + g_m i_t r_\pi)R_E$$
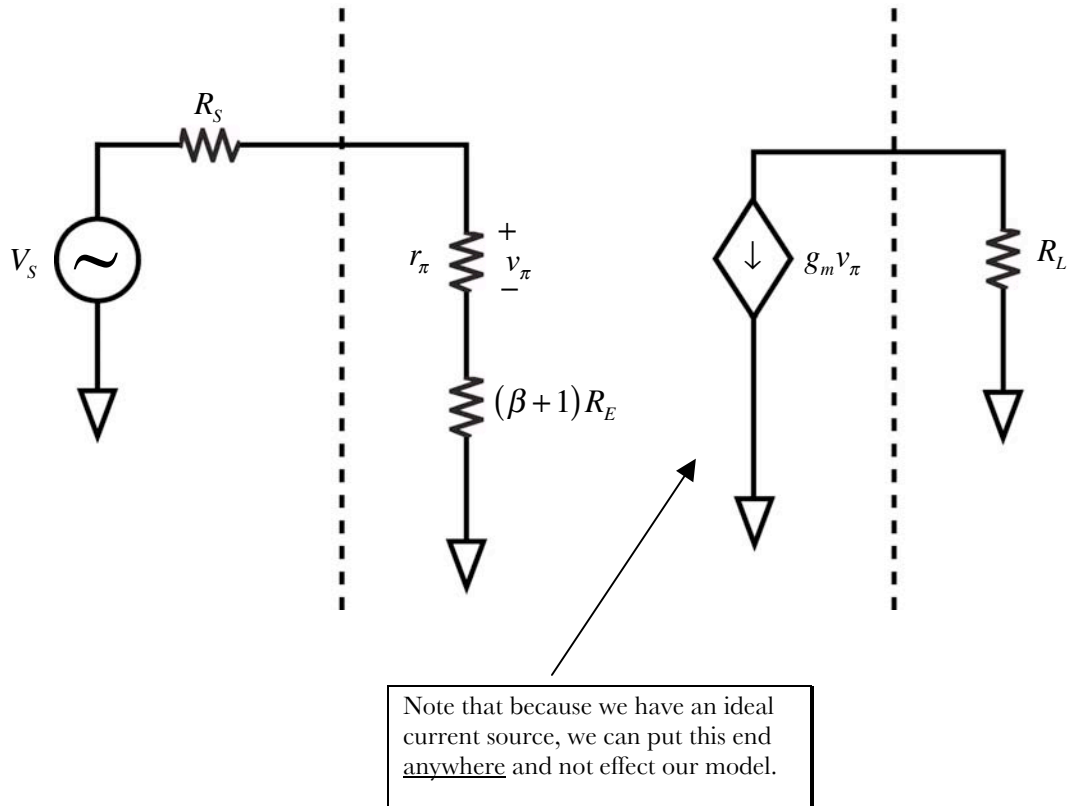$$v_t = (r_\pi + (1 + \beta)R_E)i_t$$
$$R_{IN} = r_\pi + (\beta + 1)R_E$$

So the first part of our transformation evidently looks like

Note that because we have an ideal current source, we can put this end <u>anywhere</u> and not effect our model.

What about gain? In the common emitter stage, a voltage V_i at the base resulted in $g_m V_i$ amps of current pulled through the collector. In the degenerated case:

$$v_\pi = \frac{r_\pi}{r_\pi + (\beta+1)R_E} v_i \rightarrow i_O = g_m v_\pi = \frac{g_m r_\pi}{r_\pi + (g_m r_\pi + 1)R_E} v_i$$

$$i_O = \frac{g_m}{1 + \left(g_m + \dfrac{1}{r_\pi}\right)R_E} v_i \quad \bigg| \quad \beta \gg 1 \Rightarrow g_m \gg \frac{1}{r_\pi}$$
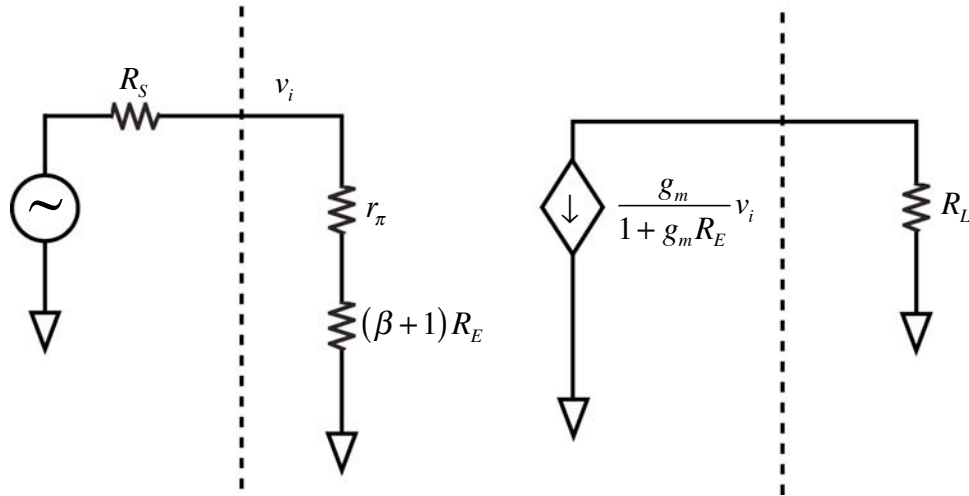
$$i_O \approx \frac{g_m}{1 + g_m R_E} v_i$$

Now we can draw our model as follows:

It required some work to get here, but this picture allows us to see things with some clarity. With regards to claims made earlier…

- Independence of stage gain: Just by looking, we can write

$$\frac{v_o}{v_i} = G_m{}' R_L = \frac{g_m R_L}{1 + g_m R_E}$$

If we choose $g_m R_E \gg 1$, $\dfrac{v_o}{v_i} \approx \dfrac{R_L}{R_E}$

- Increased Input Impedance: We went from r_π to $r_\pi + (\beta + 1) R_E$.

- Higher Bandwidth: For this we go back to the open-circuit time constant sum. Note that for $\tau_{\pi 0}$, we must go back to the original circuit:
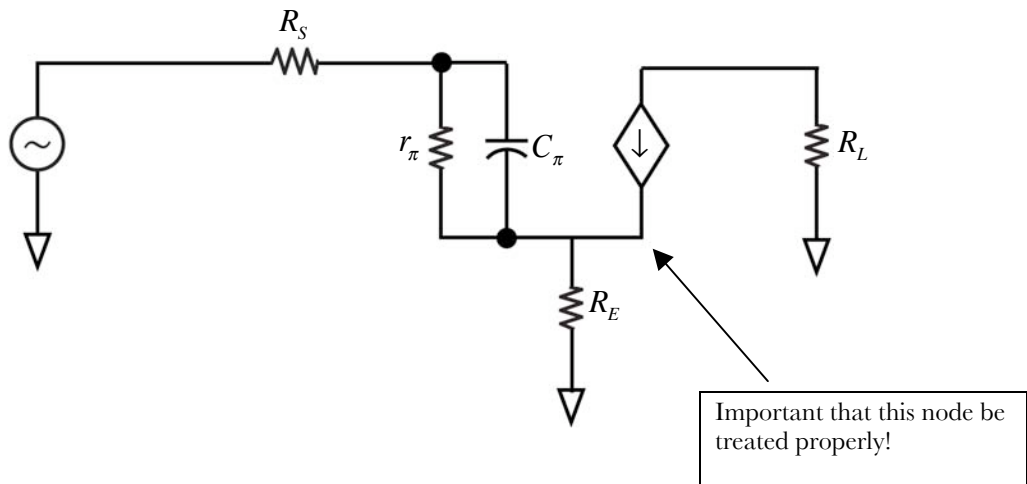
Important that this node be treated properly!

(This example sounds a cautionary note with regards to topological transformations. If you go back and look at the way we did things, you'll see that we ensured the equivalence of our model only when looking into the base or looking into the collector of the transistor. Computing $\tau_{\pi 0}$ requires access to the emitter.)
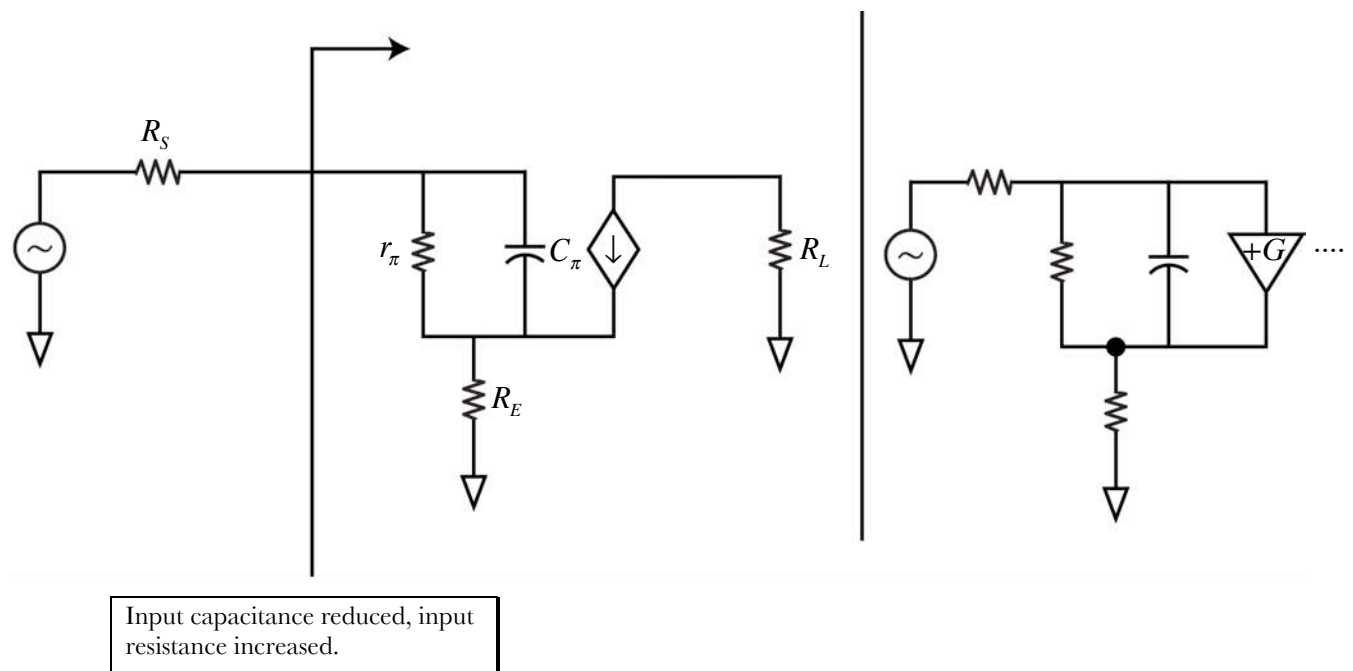
$$\tau_{\pi 0} = C_\pi \left[r_\pi \left\| \left(\frac{R_S + R_E}{1 + g_m R_E} \right) \right] < C_\pi \left[r_\pi \| R_S \right]$$

$$\tau_{\mu 0} = C_\mu \left[R_S \| \left(r_\pi + (\beta + 1) R_E \right) + R_L + \frac{g_m}{1 + g_m R_E} R_L \cdot \left(R_S \| \left(r_\pi + (\beta + 1) R_E \right) \right) \right]$$

g_m much smaller

If r_π was comparable to or bigger than R_S, this term hasn't increased by much.

So both time constants are reduced. Interestingly, the reduction in $\tau_{\pi 0}$ can be seen as yet another manifestation of the Miller Effect.
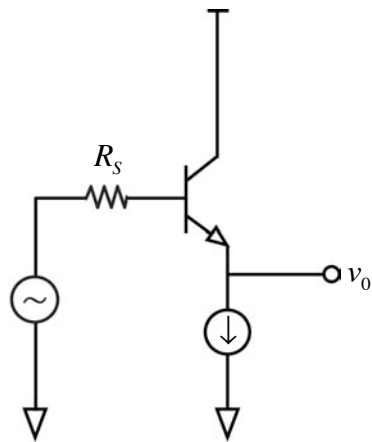

Input capacitance reduced, input resistance increased.

More on multistage amplifiers…

At the beginning of the term, we told you that you could characterize amplifier stages by their input impedance, output impedance, and gain. This is true, but gets a little more complicated when we talk about emitter followers. Consider the output impedance and input impedance

$$R_0 = R_E \left\|\left(\frac{R_S + r_\pi}{\beta + 1} \right)\right.$$

$$R_{IN} = r_\pi + (\beta + 1) R_E$$

The output impedance depends on the source resistance. Unfortunate! Nevertheless, we can still incorporate this into our model framework. A stage is now:
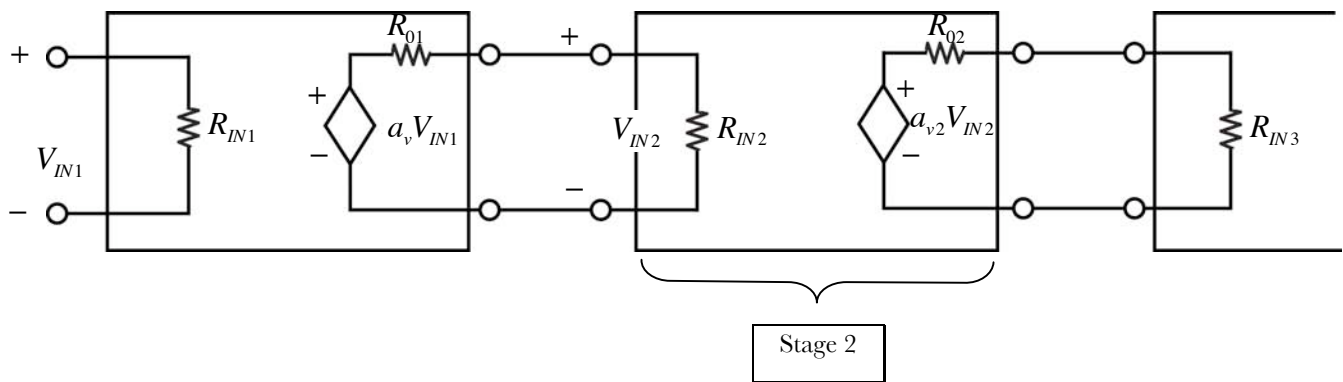
Stage 2

If stage 2 is an emitter follower, R_{IN2} is a function of R_{IN3}, and R_{O2} is a function of R_{O1}. Algebraically:

$$R_{IN2} = r_\pi + (\beta + 1)(R_E \| R_{IN3})$$

$$R_{O2} = R_E \left\|\left(\frac{R_S + r_\pi + R_{O1}}{\beta + 1} \right)\right.$$

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