DISCRETE STOCHASTIC PROCESSES
Draft of 2nd Edition

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January 31, 2011

Preface

These notes are a draft of a major rewrite of a text [9] of the same name. The notes and the text are outgrowths of lecture notes developed over some 20 years for the M.I.T. graduate subject 6.262, entitled 'Discrete Stochastic Processes.' The original motivation for 6.262 was to provide some of the necessary background for Electrical Engineering, Computer Science, and Operations Research students working in the burgeoning field of computer-communication networks. Many of the most important and challenging problems in this area involve queueing and congestion, but the literature on queueing and congestion was rapidly becoming so diffuse that a more cohesive approach was needed. Queuing problems are examples of stochastic processes, and more particularly, discrete stochastic processes. Discrete stochastic processes form a cohesive body of study, allowing queueing and congestion problems to be discussed where they naturally arise.

In the intervening years, it has become increasingly apparent that many problems involving uncertainty in almost all branches of technology and human affairs provide interesting and important examples of discrete stochastic processes. Discussing these problems as they arise both increases the application domain of this subject and also enhances our intuition and understanding about the general principles.

The purpose of this text is both to help students understand the general principles of discrete stochastic processes, and to develop the understanding and intuition necessary to apply stochastic process models to problems in engineering, science, and operations research. Although the ultimate objective is applications, there is relatively little detailed description of real applications. Rather, there are many simple examples designed both to build insight about particular principles in stochastic processes and to illustrate the generic effect of these phenomena in real systems. I am convinced that the "case study" method, in which many applications are studied in the absence of general principles, is inappropriate for understanding stochastic processes (and similarly inappropriate for any field that has a rich and highly cohesive mathematical structure).

When we try to either design new kinds of systems or understand physical phenomena, we usually employ a variety of stochastic process models to gain understanding about different tradeoffs and aspects of the problem. Creating these models requires deep understanding both of the application area and of the structure of stochastic processes. The application areas are too broad, and the structure too deep, to do all this in one text. My experience indicates that engineers rarely have difficulty applying well-understood theories and tools to well-understood application areas. The difficulty comes when the theoretical structure is not understood on both an intuitive and mathematical level. The "back of the envelope calculations" that we so prize as engineers are the result of this deep understanding of both application areas and conceptual structure.

I try to present the structural aspects of stochastic processes in the simplest possible light here, thus helping readers develop insight. This requires somewhat more abstraction than engineers are used to, but much less than often appears in mathematics texts. It also requires students to spend less time doing complex calculations and more time drawing illustrative diagrams and thinking. The proofs and explanations here are meant to be read, not because students might doubt the result, but to enhance understanding. In order to use these results in modeling real situations, the robustness of the results must be understood at an intuitive level, and this is gained only by understanding why the results are true, and why they fail when the required conditions are unsatisfied.

Students learn about new concepts in many ways, partly by learning facts, partly by doing exercises, and partly by successively refining an internal structural picture of what the subject is about. The combination of all of these leads to understanding and the ability to create models for real problems. This ability to model, however, requires much more than the "plug and chug" of matching exercises to formulas and theorems. The ability to model is based on understanding at an intuitive level, backed by mathematics.

Stochastic processes is the branch of probability dealing with probabilistic systems that evolve in time. By discrete stochastic processes, I mean processes in which changes occur only at discrete times separated by either deterministic or random intervals. In particular, we do not treat noise processes such as Gaussian processes. This distinction between discrete and non-discrete processes is somewhat artificial, but is dictated by two practical considerations. The first is that many engineering graduate students take a course involving noise, second moment theory, and inference (including detection and estimation) (the material in such subjects is more standard than the title). Such a course has much cohesion, fits nicely into one academic term, but has relatively little conceptual overlap with the material here. The second consideration is that extensions of the material here to continuous processes often obscure the probabilistic ideas with mathematical technicalities.

The mathematical concepts here are presented without measure theory, but a little mathematical analysis is required and developed as used. The material requires more patience and more mathematical abstraction than many engineering students are used to, but that is balanced by a minimum of 'plug and chug' exercises. If you find yourself writing many equations in an exercise, stop and think, because there is usually an easier way. In the theorems, proofs, and explanations, I have tried to favor simplicity over generality and clarity over conciseness (although this will often not be apparent on a first reading). I have provided references rather than proofs for a number of important results where the techniques in the proof will not be reused and provide little intuition. Numerous examples are given showing how results fail to hold when all the conditions are not satisfied. Understanding is often as dependent on a collection of good counterexamples as on knowledge of theorems. In engineering, there is considerable latitude in generating mathematical models for real problems. Thus it is more important to have a small set of well-understood tools than a large collection of very general but less intuitive results.

Most results here are quite old and well established, so I have not made any effort to attribute results to investigators, most of whom are long dead or retired. The organization of the material owes a great deal, however, to Sheldon Ross's book, Stochastic Processes, [16] and to William Feller's classic books, Probability Theory and its Applications, [7] and [8].

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Spring 2011