Problem 1 *(Iterated Elimination of Strictly Dominated Strategies)*
Consider the iterated elimination of strictly dominated strategies in the strategic form game $\langle \mathcal{I}, (S_i)_{i \in I}, (u_i)_{i \in I} \rangle$. For all $i \in \mathcal{I}$, denote the set of strategies of player i at the kth step of the elimination by S_i^k. Suppose that each $u_i(s_i, s_{-i})$ is continuous and each S_i is compact. Prove that S_i^∞ (for each i) is nonempty.

Hint: You might use the fact that intersection of nested nonempty compact sets is nonempty, i.e. Suppose $\{A_j\}$ is a collection of sets such that each A_j is nonempty, compact, and $A_{j+1} \subset A_j$. Then $A = \cap_j A_j$ is nonempty.

Problem 2 *(Iterated Elimination of Strictly Dominated Strategies in Cournot Competition)*
Consider a market in which the price charged for quantity Q of some good is given by $P(Q) = \alpha - \beta Q$ for some $\alpha, \beta > 0$. Assume that the cost of producing a unit of this good is c.

a) Assume that there are two firms in the market. Using the iterated elimination of the strictly dominated strategies construct the sets of strategies S_1^k, S_2^k for any fixed k, and conclude that S_1^∞ is a singleton. (Use the definition of S_i^k given in question 1.)

b) Assume that there are three firms. Show that S_1^∞ is not a singleton.

Problem 3 Exercise 2.1(a) from Fudenberg and Tirole.

Problem 4 *(Bertrand Competition with Different Marginal Costs)*
Suppose that two firms (A and B) produce the same good and they have strictly positive marginal costs c_A and c_B such that $c_B > c_A$. Further assume that the firms can produce as many units as they wish at those marginal costs and consumers purchase the good only if the price p offered for the good satisfies $p \leq R$ for a fixed $R > 0$.

a) Assume that if the firms offer the same price, the demand is shared equally. Show that under this tiebreaking rule there exists no pure strategy Nash equilibrium.

b) There exists a tiebreaking allocation under which the game has a unique equilibrium. Characterize this allocation and the corresponding equilibrium.

Problem 5 *(Competition with Production Constraints)*
Consider a market with 2 firms which produce the same good. Assume that the demand for this good is Q, and the consumers in this market purchase the good only if its price satisfies $p \leq R$. Further assume that the production level K of each firm satisfies $\frac{Q}{2} < K < Q$.

a) Assume that the demand is equally shared among the firms when they offer the same price. Under this tiebreaking rule write the payoff functions of the firms

b) Show that there does not exist a pure strategy Nash equilibrium under this tiebreaking rule.

c) Prove that this result does not depend on the tiebreaking rule.

Problem 6 *(A war of attrition)* Two players are involved in a dispute over an object. The value of the object to player i is $v_i > 0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the first player to concede does so at time t, the other player obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player i receiving a payoff of $v_i/2$. Time is valuable: until the first concession each player loses one unit of payoff per unit of time.

Formulate this situation as a strategic game and show that in all Nash equilibria one of the players concedes immediately.