Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

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Problem Set 5 (due March 10, 2004) [1]

Problem 5.1

Use KYP Lemma to find (analytically) the set of all $a \in \mathbf{R}$ such that the Riccati equation

$$PA + A'P = (C' - PB)(C - B'P),$$

where (A, B) is controllable, (C, A) is observable, and

$$C(sI - A)^{-1}B = (s + a)^{-1000},$$

has a stabilizing solution $P = P'$.

Problem 5.2

Using the generalized Parrot's theorem, write down an algorithm for finding matrix L which minimizes the largest eigenvalue of

$$M = M(L) = \begin{bmatrix} \alpha & \beta + 2L \\ 2L' + \beta' & \gamma + L'L \end{bmatrix},$$

where $\alpha = \alpha'$, β, and $\gamma = \gamma'$ are given matrices.

[1] Version of March 3, 2004

Problem 5.3

Use the KYP Lemma to write a MATLAB algorithm for checking that a given stable transfer function $G = G(s)$, available in a state space form, satisfies the condition

$$|G(j\omega)| > 1 \quad \forall\, \omega \in \mathbf{R} \cup \{\infty\}.$$

The algorithm should be exact, provided that the linear algebra operations involved (matrix multiplications, eigenvalue calculations, comparison of real numbers) are performed without numerical errors. In particular, checking that $|G(j\omega_k)| > 1$ at a finite set of frequencies ω_k is not acceptable in this problem[2].

[2]Of course, frequency sampling may be acceptable in many practical applications