Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.061/6.690 Introduction to Power Systems

Problem Set 6 Solutions March 30, 2011

Problem 1: Problem 8 from Chapter 7

Put this one on a 100 MVA base. The impedances are: Generator: $x_g = \frac{100}{500} \times .25 = .05$
Transformer: $x_t = \frac{100}{500} \times .05 = .01$
Line base impedance is $Z_B = \frac{345^2}{100} = 1190\Omega$, so 50 km of line has per-unit impedance: $z_\ell = j.0147 + .0008$

The problem can be represented as shown in the circuit diagram of Figure 1. The generator and transformer are lumped together to form a reactance of 0.6 per-unit. The upper line and right-hand part of the lower line are in series with an impedance of three times the left-hand side of the lower line. Total impedance from the source to the fault is: $z = j.06 + z_\ell || 3z_\ell \approx j.071 + .0006$. Currents through the two line segments are determined by a current divider:

$$i_1 = \frac{1}{4} i_F$$
$$i_2 = \frac{3}{4} i_F$$

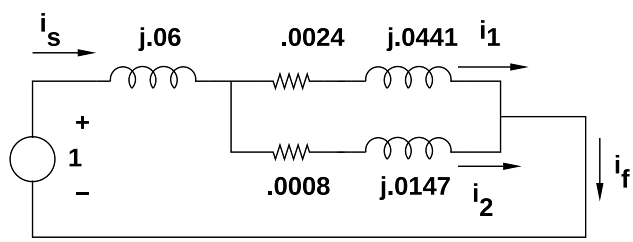

Figure 1: Fault Situation

Then the per-unit currents are:

$$i_F = \frac{1}{j.071 + .0006} \approx .12 - j14.2$$
$$i_1 = \frac{1}{4} i_F \approx .03 - j3.55$$
$$i_2 = \frac{3}{4} i_F \approx .09 - j10.65$$

To convert to ordinary variables, we need base currents:

$$I_{BH} = \frac{100}{\sqrt{3}345} = 167\text{A}$$
$$I_{BG} = \frac{100}{\sqrt{3}24} = 2406\text{A}$$

Then the currents are:

fault	(345 kV)	$20 - j2371$A
transformer	(345 kV)	$20 - j2371$A
upper line	(345 kV)	$5 - j593$A
lower left line	(345 kV)	$15 - j1779$A
generator	(24 kV)	$289 - j34165$A

Problem 2: Problem 11 from Chapter 7

GMD of the bundles is: GMD $= \sqrt{.78 \times .03 \times .5} \approx .108$m

$$
\begin{aligned}
(L-M)_{\text{adjacent}} &= \frac{\mu_0}{2\pi} \times \log \frac{10}{.14} \approx 9.06 \times 10^{-7}\text{H/m} \\
(L-M)_{\text{outside}} &= \frac{\mu_0}{2\pi} \times \log \frac{20}{.14} \approx 10.04 \times 10^{-7}\text{H/m} \\
(L-M)_{\text{average}} &= \frac{2}{3}(L-M)_{\text{adjacent}} + \frac{1}{3}(L-M)_{\text{outside}} \approx 9.5 \times 10^{-7}\text{H/m}
\end{aligned}
$$

Resistance of the aluminum conductors is: $R = \frac{1}{2}\frac{1}{\pi \times .03^2 \times 3 \times 10^7} \approx 5.895 \times 10^{-6}\Omega/\text{m}$

Then, since 10 km is 10^4 m, $Z = .05985 + j3.582\Omega$

Problem 3: Problems 1 and 2 from Chapter 10

part a : $I_1 = \frac{I}{3}$, $I_2 = \frac{I}{3}$, $I_0 = \frac{I}{3}$

part b : $I_1 = \frac{I}{3}a$, $I_2 = \frac{I}{3}a^2$, $I_0 = \frac{I}{3}$

part c : $I_1 = \frac{j}{\sqrt{3}}I$, $I_2 = -\frac{j}{\sqrt{3}}I$, $I_0 = 0$

positive sequence : $I_a = I$, $I_b = a^2 I$, $I_c = aI$

negative sequence : $I_a = I$, $I_b = aI$, $I_c = a^2 I$

zero sequence : $I_a = I$, $I_b = I$, $I_c = I$

Phasor diagrams are in Figure 2.

Problem 4: Problem 8 from Chapter 10

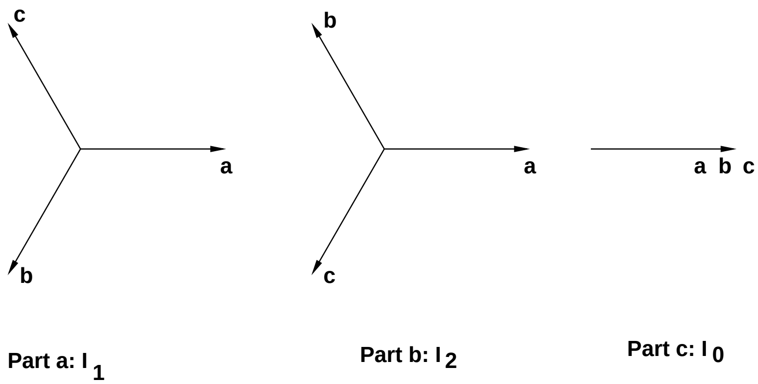

Figure 2: Phasor Diagrams for Chapter 10, Problem 2

If the star point is *un*grounded, its voltage is:

$$V_n = V_a \frac{R_b||R_c}{R_a + R_b||R_c} + V_b \frac{R_a||R_c}{R_b + R_a||R_c} + V_c \frac{R_a||R_b}{R_c + R_a||R_b}$$

Taking advantage of the b-c symmetry:

$$V_n = V_a \left(\frac{R_b||R_c}{R_a + R_b||R_c} - \frac{R_a||R_c}{R_b + R_a||R_c} \right) = 277.1 \times \left(\frac{6}{16} - \frac{5.54}{17.5} \right) \approx 0.063 \times 277.1 \approx 17.4\text{V}$$

Then the three phase currents are:

$$\begin{aligned} I_a &= \tfrac{277.1-17.4}{10} = 25.97 = & 23.09 + 2.88\text{A} \\ I_b &= \tfrac{277.1e^{-j\frac{2\pi}{3}}-17.4}{12} = & 23.09e^{-j\frac{2\pi}{3}} - 1.45\text{A} \\ I_c &= \tfrac{277.1e^{j\frac{2\pi}{3}}-17.4}{12} = & 23.09e^{j\frac{2\pi}{3}} - 1.45\text{A} \end{aligned}$$

Then the symmetrical components are:

$$\begin{aligned} I_1 &= \tfrac{1}{3}\left(I_a + aI_b + a^2 I_c\right) = 23.09 + \tfrac{1}{3}(2.88 + 1.45) \approx & 24.53\text{A} \\ I_2 &= \tfrac{1}{3}(2.88 + 1.45) \approx & 1.44\text{A} \\ I_0 &= & 0 \end{aligned}$$

Problem 5: Problem 16 from Chapter 10

This one is solved by the script that is appended. The solution is in the output of that script is:

```
Problem 10_16
Base Currents: Generator 4183.7  Line 418.4
Per-Unit Currents, line-neutral in Phase a
```

```
Fault i_a = 0.000+j -2.727
Generator i_a = 0.000+j -1.575  i_b = 0.000+j 1.575  i_c = 0.000+j -0.000
Currents in Amperes\
Fault I_a = 1141.0  Ib = 0.0  Ic =  0.0
Generator I_a = 6587.6  I_b = 6587.6  I_c = 0.0

Per-Unit Currents, line-line in Phases b and c
Fault i_a = 0.000+j 0.000 i_b = -2.038+j 0.000  i_c = 2.038+j 0.000
Generator i_a = -1.176+j 0.000  i_b = -1.176+j 0.000  i_c = 2.353+j 0.000
Currents in Amperes
Fault I_a = 0.0  Ib = 852.5  Ic = 852.5\
Generator I_a = 4922.0  I_b = 4922.0  I_c = 9844.0
```

The script for this problem is:

```
% Solution: Chapter 10, Problem 16

% Base Quantities
P_B = 100;               % MVA
V_Bg = 13.8;             % kV
V_Bl = 138;              % kV
I_Bg = 1000*P_B/(sqrt(3)*V_Bg);
I_Bl = 1000*P_B/(sqrt(3)*V_Bl);

% per-unit impedances
z_g1 = j*.25;
z_g2 = j*.25;
z_l1 = j*.125;
z_l2 = j*.125;
z_l0 = j*.2;
z_t1 = j*.05;

% line-neutral fault
z1 = z_g1 + z_l1 + z_t1;  % sequence impedances
z2 = z1;
z0 = z_t1 + z_l0;
i_f = 1/(z1+z2+z0);      % fault current

% currents at the fault: are also currents in the line
i_af = 3*i_f;
i_bf = 0;
i_cf = 0;

rl = exp(j*pi/6);        % rotation by   30 degrees
rr = exp(-j*pi/6);       % rotation by  -30 degrees
```

```
r2l = exp(j*pi/3);        % rotation by   60 degrees
r2r = exp(-j*pi/3);       % rotation by  -60 degrees
a = exp(j*2*pi/3);        % rotation by  120 degrees
a2 = exp(-j*2*pi/3);      % rotation by -120 degrees

% currents in the generator leads
i_ag = i_f * (rr + rl);
i_bg = i_f * (rr*a2 + rl*a);
i_cg = i_f * (rr*a + rl*a2);

% in amps
I_af = i_af * I_Bl;
I_bf = 0;
I_cf = 0;

I_ag = i_ag * I_Bg;
I_bg = i_bg * I_Bg;
I_cg = i_cg * I_Bg;

fprintf('Problem 10_16\n')
fprintf('Base Currents: Generator %10.1f  Line %10.1f \n', I_Bg, I_Bl)
fprintf('Per-Unit Currents, line-neutral in Phase a\n')
fprintf('Fault i_a = %10.3f+j%10.3f \n', real(i_af), imag(i_af))
fprintf('Generator i_a = %10.3f+j%10.3f  i_b = %10.3f+j%10.3f  i_c = %10.3f+j%10.3f\n', re
    imag(i_bg), real(i_cg), imag(i_cg))
fprintf('Currents in Amperes\n')
fprintf('Fault I_a = %10.1f  Ib = %10.1f  Ic = %10.1f\n', abs(I_af), abs(I_bf), abs(I_cf))
fprintf('Generator I_a = %10.1f  I_b = %10.1f  I_c = %10.1f\n', abs(I_ag), abs(I_bg), abs(

% now for line-line fault
z1 = z_g1 + z_l1 + z_t1;  % sequence impedances
z2 = z1;

i_f = 1/(z1 + z2);
i_1 = i_f;
i_2 = - i_f;
i_0 = 0;

% at the fault
i_af = i_1 + i_2;
i_bf = a2*i_1 + a*i_2;
i_cf = a*i_1 + a2*i_2;

% in the generator leads
i_ag = rr*i_1 + rl*i_2;
```

```
i_bg = rr*a2*i_1 + rl*a*i_2;
i_cg = rr*a*i_1 + rl*a2*i_2;
% in amps
I_af = i_af * I_Bl;
I_bf = i_bf * I_Bl;
I_cf = i_cf * I_Bl;

I_ag = i_ag * I_Bg;
I_bg = i_bg * I_Bg;
I_cg = i_cg * I_Bg;

fprintf('Problem 10_16\n')
fprintf('Per-Unit Currents, line-line in Phases b and c\n')
fprintf('Fault i_a = %10.3f+j%10.3f i_b = %10.3f+j%10.3f  i_c = %10.3f+j%10.3f\n', real(i_
    real(i_bf), imag(i_bf), real(i_cf), imag(i_cf))
fprintf('Generator i_a = %10.3f+j%10.3f  i_b = %10.3f+j%10.3f  i_c = %10.3f+j%10.3f\n', re
    imag(i_bg), real(i_cg), imag(i_cg))
fprintf('Currents in Amperes\n')
fprintf('Fault I_a = %10.1f  Ib = %10.1f  Ic = %10.1f\n', abs(I_af), abs(I_bf), abs(I_cf))
fprintf('Generator I_a = %10.1f  I_b = %10.1f  I_c = %10.1f\n', abs(I_ag), abs(I_bg), abs(
```

Problem 6: For 6.690

The way of working this problem is to find the mutual admittance between positive sequence voltage and negative sequence current. There will be positive sequence voltage applied across the combined impedance of the line, transformers and generators because the system is carrying current (in fact it is carrying real power), and then that voltage, applied to the admittance matrix for the system results in current in both positive and negative sequences. The transmission line has phase admittance:

$$\underline{\underline{Z}}_{ph} = j377 \times \begin{bmatrix} .012 & .006 & .003 \\ .006 & .012 & .006 \\ .003 & .006 & .012 \end{bmatrix}$$

That impedance is normalized using the line base impedance (see Problem 2) and then the positive and negative sequence impedances are added. To find the voltage applied to the line we see we are carrying some real power (If you normalized to 100 MVA that would be 2 per-unit). Voltage across the line is just the difference between the sending and receiving ends and that is applied to the admittance of the system.

A script to implement this procedure was written and is appended. The results are below:

```
Problem 6.6
Transmission Line Impedance, per-unit

zl =
```

```
        0 + 0.0044i        0 + 0.0025i        0 + 0.0013i
        0 + 0.0025i        0 + 0.0044i        0 + 0.0025i
        0 + 0.0013i        0 + 0.0025i        0 + 0.0044i

Line Symmetrical Component Impedances

zls =

   0.0000 + 0.0023i  -0.0007 + 0.0004i  -0.0004 - 0.0002i
   0.0007 + 0.0004i   0.0000 + 0.0023i   0.0004 - 0.0002i
   0.0004 - 0.0002i  -0.0004 - 0.0002i  -0.0000 + 0.0087i

Total Admittances

yt =

   0.0000 - 5.7202i  -0.0239 + 0.0138i   0.0000 - 0.0000i
   0.0239 + 0.0138i   0.0000 - 5.7202i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 - 0.0000i   0.0000 - 0.0000i

Phase Angle = 20.4657 degrees
Currents

I =

   0.0000 - 2.0324i
   0.0085 + 0.0049i
   0.0000 + 0.0000i

Per-Unit I_2 = 0.00982127Problem 6.3
Transmission Line Impedance, per-unit

zl =

        0 + 0.0038i        0 + 0.0019i        0 + 0.0010i
        0 + 0.0019i        0 + 0.0038i        0 + 0.0019i
        0 + 0.0010i        0 + 0.0019i        0 + 0.0038i

Line Symmetrical Component Impedances

zls =

   0.0000 + 0.0022i  -0.0005 + 0.0003i  -0.0003 - 0.0002i
   0.0005 + 0.0003i   0.0000 + 0.0022i   0.0003 - 0.0002i
```

```
   0.0003 - 0.0002i  -0.0003 - 0.0002i  -0.0000 + 0.0070i

Total Admittances

yt =

   0.0000 - 5.7236i  -0.0180 + 0.0104i   0.0000 - 0.0000i
   0.0180 + 0.0104i   0.0000 - 5.7236i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 - 0.0000i   0.0000 - 0.0000i

Phase Angle = 20.4528 degrees
Currents

I =

   0.0000 - 2.0323i
   0.0064 + 0.0037i
   0.0000 + 0.0000i

Per-Unit I_2 = 0.00737018
```

The per-unit negative sequence current is about three quarters of one percent, on the 100 MVA base. Converted to the 400 MVA base of the generator this would be about .00185 of one percent.

Appendix: Script for Problem 6

```
% Problem 6.6: Unbalanced transmission line
a = exp(j*2*pi/3);                                                % housekeeping stuff
T = (1/3) .* [1 a a^2;1 a^2 a; 1 1 1];
TI = inv(T);
Zl = j*377 .* [.012 .006 .003;.006 .012 .006; .003 .006 .012];   % line impedance (phase)
zl = Zl ./ 1190;                                                  % normalize
zls = T*zl*TI;                                                    % transform to sequence
z1 = j*.045+j*.0125+j*.025+j*.09;                                 % rest of network
ze = [z1 0 0;0 z1 0;0 0 10^6];                                    % balanced part: zero seq is open
zt = zls+ze;                                                      % to get driving voltage
yt = inv(zt);
x =imag(zt(1,1));                                                 % here is end to end reactance
delt = asin(2*x);                                                 % required phase shift
v = 2*sin(delt/2);                                                % and resulting voltage
V = [v; 0; 0];                                                    % assumed to be positive sequence
I = yt*V;                                                         % and here are resulting currents
fprintf('Problem 6.3\n')
fprintf('Transmission Line Impedance, per-unit\n')
zl
fprintf('Line Symmetrical Component Impedances\n')
zls
fprintf('Total Admittances\n');
yt
fprintf('Phase Angle = %g degrees\n', (180/pi)*delt)
fprintf('Currents\n')
I
fprintf('Per-Unit I_2 = %g\n', abs(I(2)));
```

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