

Problem Set 1

This problem set is due **at 9:00pm on Wednesday, February 15, 2012.**

Both exercises and problems should be solved, but *only the problems* should be turned in. Exercises are intended to help you master the course material. Even though you should not turn in the exercise solutions, you are responsible for material covered by the exercises.

Mark the top of the first page of your solution with your name, the course number, the problem number, your recitation section, the date, and the names of any students with whom you collaborated. The homework template (L^AT_EX) is available on the course website.

You will often be called upon to “give an algorithm” to solve a certain problem. Your write-up should take the form of a short essay. A topic paragraph should summarize the problem you are solving and what your results are. The body of the essay should provide the following:

1. A description of the algorithm in English and, if helpful, pseudo-code.
2. At least one worked example or diagram to show more precisely how your algorithm works.
3. A proof (or indication) of the correctness of the algorithm.
4. An analysis of the running time of the algorithm.

Remember, your goal is to communicate. Full credit will be given only to correct solutions *that are described clearly*. Convolved and opaque descriptions will receive lower marks.

Exercise 1-1. Do Exercise 2.3-3 in CLRS.

Exercise 1-2. Do Exercise 2.3-4 in CLRS.

Exercise 1-3. Do Exercise 3.1-2 in CLRS.

Exercise 1-4. Do Exercise 3.1-3 in CLRS.

Exercise 1-5. Do Exercise 3.1-4 in CLRS.

Exercise 1-6. Do Exercise 3.2-3 in CLRS.

Exercise 1-7. Do Exercise 9.3-1 in CLRS.

Exercise 1-8. Do Exercise 9.3-2 in CLRS.

Problem 1-1. Variations of Median Select

Recall from lecture the worst case linear time algorithm for selecting the i th smallest element from a set S of n distinct elements. In class we organized the elements into groups of 5, in this problem we will analyze the behavior of the algorithm by picking different sizes for the groups.

- (a) If we choose the group sizes to be 7, and picking the median element from each group of 7, repeat the analysis done in CLRS and write down the recurrence relation for $T(n)$. Solve the recurrence using the substitution method and prove its correctness using induction, in particular, state the assumptions you make about the base cases (for example, for groups of 5, we assumed $T(n) \leq cn$ for $n \leq 140$).
- (b) What happens if we choose the group size to be 4? Since the median of 4 elements is somewhat ambiguous, assume that we always choose the *lower median* from every group, that is, the 2nd smallest element. Again, write down the recurrence relation for $T(n)$, solve it using the substitution method and prove its correctness using induction. Again stating your assumptions.

Problem 1-2. Land for sale in one dimension

- (a) In 1-D land, everybody lives in a one dimensional region. You own a stretch of land in the shape of a circle, and would like to sell it to n customers. Each of these customers is interested in a contiguous segment on the perimeter (ie. an arc), which can be represented in polar coordinates. For example, customer i wants to have θ_{i1} to θ_{i2} on the circle. Unfortunately, many of these arcs overlap, and you cannot sell a portion of a customer's request. Devise an algorithm to maximize the number of customers' requests you can fulfill.
- (b) You are a firm believer in the free market and competition. So, in addition to each customer giving you an arc that they would like, he also offers a price. However, you still cannot sell a portion of any request. Devise an algorithm to maximise your profit.
- (c) You have acquired a new area in the shape of a cross. Once again, there are n customers wanting to pay a price for a contiguous 1-D region on the cross (which could be a line segment, a "T" segment, or a "cross" segment). Like before, these requested regions overlap and can only be sold in its entirety. Devise an algorithm to maximize your profit.

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