

## 6.045 Pset 2

Assigned: Friday, February 11, 2011

Due: Thursday, February 24, 2011

**To facilitate grading, remember to solve each problem on a separate sheet of paper!**

1. Show that the following languages are context-free.
  - (a)  $L = \{a^n b^{2n} : n \geq 0\}$
  - (b) The language  $L \subset \{(,)\}^*$  that consists of all strings of *balanced parentheses*: for example,  $((())())$  is in  $L$ , but  $()()$  is not in  $L$ .
  - (c)  $L = \{x \in \{a, b\}^* \mid x \text{ contains an equal number of } a\text{'s and } b\text{'s}\}$
  - (d)  $L = \{x \in \{a, b\}^* \mid x \text{ contains more } a\text{'s than } b\text{'s}\}$
  - (e) [Extra credit]  $L = \{x\#y \mid x, y \in \{a, b\}^* \text{ and } x \neq y\}$
2. Show that context-free languages are *closed under union*: that is, if  $A$  and  $B$  are both CFLs, then  $A \cup B$  is a CFL also.
3. Show that every regular language is also a CFL. [Hint: Explain how to convert any regular expression into a CFG that generates the same language.]
4. Let  $L_1 = \{a^n b^n c^m \mid n, m \geq 0\}$  and  $L_2 = \{a^n b^m c^m \mid n, m \geq 0\}$ .
  - (a) Show that  $L_1$  and  $L_2$  are both CFLs. [Note: You only need to give a CFG generating  $L_1$ ; for  $L_2$  you can appeal to the symmetry with  $L_1$ .]
  - (b) Recall from pset1 that regular languages are *closed under intersection*: that is, if  $A$  and  $B$  are both regular, then so is  $A \cap B$ . Using problem 4a together with a result from class, show that CFLs are *not* similarly closed under intersection.
  - (c) Show that CFLs are not *closed under complement*: that is, even if  $L$  is a CFL, the complementary language  $\overline{L} = \{x \mid x \notin L\}$  need not be a CFL. [Hints: problem 2,  $L_1$  and  $L_2$ , De Morgan's Law.]
5. Let  $L$  be language consisting of 1, 101, 101001, 1010010001, etc. Show that  $L$  is not context-free.
6. Let  $L = \{1^n \mid n \text{ is prime}\}$ .
  - (a) Show that  $L$  is not regular. (You can use the fact that there are infinitely many prime numbers.)
  - (b) Show that the regular languages are closed under complement. Conclude that  $\overline{L} = \{1^n \mid n \text{ is composite}\}$  is not regular either.
  - (c) Show that  $L$  is not context-free.
  - (d) [Extra credit] Show that  $\overline{L} = \{1^n \mid n \text{ is composite}\}$  is not context-free. (You can use Dirichlet's Theorem, which says that if  $\text{GCD}(n, k) = 1$ , then the sequence  $n, n + k, n + 2k \dots$  contains infinitely many primes.)

7. Let  $L = \{\#x\# \mid x \in \{0,1\}^* \text{ is a palindrome}\}$ . Design a Turing machine, over the alphabet  $\{0, 1, \#\}$ , that recognizes  $L$ . Give the complete state transition diagram.
8. Let  $L = \{\#x\# \mid x \in \{0,1\}^* \text{ contains an equal number of 0's and 1's}\}$ . Verbally describe a Turing machine, over the alphabet  $\{0, 1, \#\}$ , that recognizes  $L$ . (*Note:* You can't just write "count the number of 0's and 1's"—explain how the counting is done!)

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6.045J / 18.400J Automata, Computability, and Complexity  
Spring 2011

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