PROFESSOR: Independent events are events that have nothing to do with each other. And needless to say, it's a lot easier to work with them because you don't have to figure out some weird interaction between two events that do have something to do with each other.

Typical case where independent events come up is, for example, you toss a coin five times, and then you're about to toss a coin the sixth time. Is there any reason to think that what the coins did the first five times is going to have any influence on the flip of the coin for the sixth time? Well, it's reasonable to assume not, which is to say that the outcome of the sixth toss is arguably independent of the outcome of all the previous tosses.

OK. Let's look at a formal definition now in this short video of just what is the technical definition of independent events.

So what we said is that they are independent if they have nothing to do with each other. What that means is that if I tell you that B happened, it doesn't have any effect on the probability of A. That is, the probability of A, given that B happened, doesn't change the probability of A at all. That's it. Now this is one definition. Maybe this is the more intuitive definition.

But another definition that's equivalent and is standard is that two events are equivalent if the product of their probabilities is equal to the probability that they both happen, that is, the probability of their intersection. Now definition one and definition two are trivial equivalent, just using the definition of conditional probability.

And if you don't see that, this would be a moment to stop, get a pencil and paper, and write down the three-line proof of the equivalence of these two equalities. In fact, the three-line proof has this as the first line and that as the second line. So you could argue it's really just the middle line that you're adding. OK.

Definition two has the slight advantage that it always works, whereas definition one implicitly requires that the divisor-- remember probability of A given B is defined as the probability of the intersection divided by the probability B. It's only defined if the probability of B is positive. Whereas the second definition always works, so we don't have to put a proviso in about the probability of B being non-zero. So that's the definition of independence.

Looking at this definition, what you can see immediately is that it's completely symmetric in A and B. Since multiplication is commutative and intersection is commutative, which is A and which is B doesn't matter. And what that implies then is that A is independent of B if and only if B is independent of A.

Now another fact that holds is that if the probability of B happens to be zero, then vacuously B is independent of everything-- even itself. Which isn't important, but is a small technicality that's worth remembering by that definition.

Now again, the intuitive idea that A and B have nothing to do with each other is that A is independent of B means that A is independent of whether or not B occurs. That is to say, if A is independent of B, it ought to be independent of the complement of B.

And that's a lemma that's also easily proved. A is independent of B if and only if A is independent of the complement of B. It's a simple proof using the difference rule. And again, I encourage you to stop with a piece of paper and a pencil and convince yourself that that follows with a one-line proof.