Mathematics for Computer Science

6.042J/18.062J

WELCOME!

Prof. Albert R. Meyer
Prof. Ronitt Rubinfeld

"Proof, Proofs & More Proofs"

Quick Summary

1. Fundamental Concepts of Discrete Mathematics.
2. Discrete Mathematical Structures

   (like *trees* or *lists*)
3. Discrete Probability Theory.

Vocabulary

Quickie:

What does "discrete" mean?

(≠ "discreet")

Online Tutor Problems 1

Due Friday, 1pm:

Part 1.1: Course Registration

Due Monday, 1pm:

Part 1.2: Diagnostic Questionnaire

Reading Assignment

Reading: see course calendar

Email comments:

due Wednesday 11am

Course Organization

- Web site: All course handouts.
- Problem Sets: *up to 30%* of grade (see course info).

Course Organization

- Studio-Lecture Style: mix of mini-lectures & team problem-solving; preparation & attendance required (25% of grade)

Active Lectures

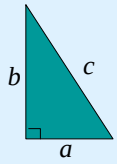
Say "hello" to your neighbors -- you'll be working with them .

Active Lectures

Quickie question:

Where was your neighbor born?

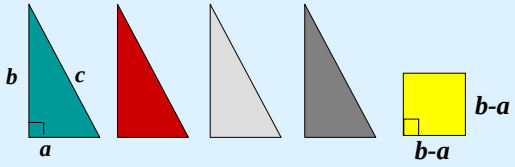
Getting started: Pythagorean theorem



$$a^2 + b^2 = c^2$$

Familiar? Yes!
Obvious? No!

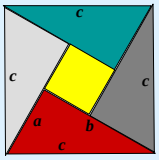
A Cool Proof



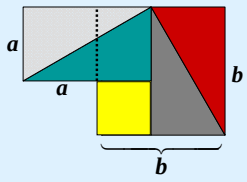
Rearrange into: (i) a $c \times c$ square, and then
(ii) an $a \times a$ & a $b \times b$ square

(Many many proofs: `http://www.cut-the-knot.com`)

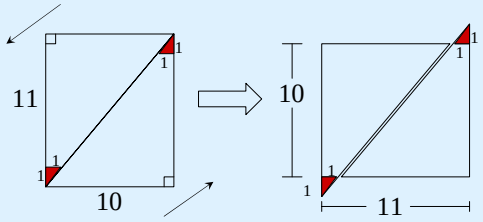
A Cool Proof



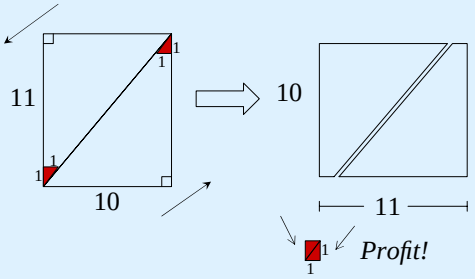
A Cool Proof



A False Proof: Getting Rich By Diagram



A False Proof: Getting Rich By Diagram

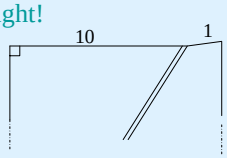


Getting Rich

The bug:

are not right triangles!

The top and bottom line of the "rectangle" is not straight!



Another False Proof

Theorem:

Every polynomial, $ax^2 + bx + c$, has two roots over $\mathbb{C}$.

Proof (by calculation):

The polynomial $ax^2 + bx + c$ has roots

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Another False proof

Counter-examples:

$0x^2 + 0x + 1$ has 0 roots.

$0x^2 + 1x + 1$ has 1 root.

The bug: divide by zero error.

The fix: assume $a \neq 0$.

Another false proof

Counter-example:

$1x^2 + 0x + 0$ has 1 root.

The bug: $r_1 = r_2$

The fix: need hypothesis $D \neq 0$ where

$$D ::= \sqrt{b^2 - 4ac}$$

Another false proof

Ambiguity when $D < 0$:

$x^2 + 1$ has roots i, $-i$.

Which is r_1, which is r_2?

1 = -1 ?

The ambiguity causes problems:

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = \left(\sqrt{-1}\right)^2 = -1$$

Moral: "mindless" calculation not safe.

1. Be sure rules are properly applied.
2. Calculation is a risky substitute for understanding.

Consequences of 1= –1

$\frac{1}{2} = -\frac{1}{2}$ (multiply by $\frac{1}{2}$)

$2 = 1$ (add $\frac{3}{2}$)

"Since I and the Pope are clearly 2*, we conclude that*
I and the Pope are 1*.*
That is, I am the Pope."
-- Bertrand Russell

Consequences of 1= –1

(Picture source: http://www.users.drew.edu/~jlenz/brs.html)

Bertrand Russell (1872 - 1970)

In-class Problems

PROBLEMS 1 & 2