6.041SC Probabilistic Systems Analysis and Applied Probability, Fall 2013 Transcript – Tutorial: Normal Probability Calculation

Hi. In this video, we're going to do standard probability calculations for normal random variables. We're given that x is standard normal with mean 0 and variance 1. And y is normal with mean one and variance 4. And we're asked for a couple of probabilities.

For the normal CDF, we don't have a closed form expression. And so people generally tabulate values and for the standard normal case. So if we want little x equal to 3.49, we just look for 3.4 along the rows and 0.09 along the columns, and then pick the value appropriately. So for part A, we're asked what's the probability that x is less than equal to 1.5?

That's exactly phi of 1.5 and we can look that up. 1.5 directly and that's 0.9332. Then were asked, what's the probability that x is less than equal to negative 1? Notice that negative values are not on this table. And the reason that is is because the standard normal is symmetric around zero. And we don't really need that.

We just recognize that the area in this region is exactly the area in this region. And so that's equal to the probability that x is greater than equal to 1. This is equal to 1 minus the probability that x is less than 1. And we can put the equal sign in here because x is continuous, it doesn't matter.

And so we're going to get, this is equal to 1 minus phi of one. And we can look up phi of 1, which is 1.00, and that's 0.8413. OK.

For part B, we're asked for this distribution of y minus 1 over 2. So any linear function of a normal random variable is also normal. And you can see that by using the derived distribution for linear functions of random variables.

So in this case, we only need to figure out what's the mean and the variance of this normal random variable. So the mean in this case, I'm going to write that as y over 2 minus 1/2. The expectation operator is linear and so that's going to be-- and the expectation in this case is 1, so that's going to be 0.

Now the variance. For the shift, it doesn't affect the spread. And so the variance is exactly going to be the same without the minus 1/2. And for the constant, you can just pull that out and square it. And the variance of y we know is 4. And so that's 1/4 times 4, that's 1.

OK. So now we know that y minus 1 over 2 is actually standard normal. Actually for any normal random variable, you can follow the same procedure. You just subtract its mean, which is 1 in this case. And divide by its standard deviation and you will get a standard normal distribution.

All right, so for part C we want the probability that y is between negative 1 and 1. So let's try to massage it so that we can use the standard normal table. And we already know that this is standard normal, so let's subtract both sides by negative 1. And that's equal to-- I'm going to call

this standard normal z, so that's easier to write. And that's equal to negative 1 less than equal to z, less than equal to zero. So we're looking for this region, 0, 1, negative 1. So that's just the probability that it's less than zero minus probability that it's less than negative 1.

Well for a standard normal, half the mass is below zero and a half the mass is above. And so that's just going to be 0.5 directly. And for this, we've already computed this for a standard normal, which was x in our case. And that was 1 minus 0.8413. Done.

So we basically calculated a few standard probabilities for normal distributions. And we did that by looking them up from the standard normal table.

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