

Recitation 7
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1. **Problem 2.35, page 130 in the text.** Verify the expected value rule

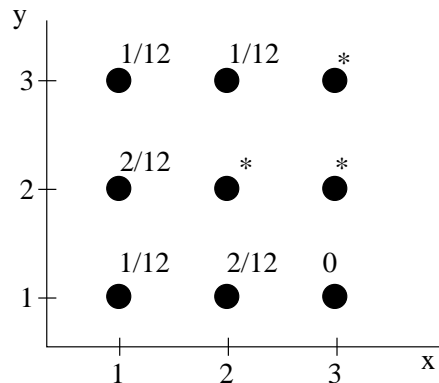
$$\mathbf{E}[g(X, Y)] = \sum_x \sum_y g(x, y)p_{X,Y}(x, y),$$

using the expected value rule for a function of a single random variable. Then, use the rule for the special case of a linear function, to verify the formula

$$\mathbf{E}[aX + bY] = a\mathbf{E}[X] + b\mathbf{E}[Y],$$

where a and b are given scalars.

2. Random variables X and Y can take any value in the set $\{1, 2, 3\}$. We are given the following information about their joint PMF, where the entries indicated by a * are left unspecified:



- (a) What is $p_X(1)$?
- (b) Provide a clearly labeled sketch of the conditional PMF of Y given that $X = 1$.
- (c) What is $\mathbf{E}[Y | X = 1]$?
- (d) Is there a choice for the unspecified entries that would make X and Y independent?

Let B be the event that $X \leq 2$ and $Y \leq 2$. We are told that conditioned on B , the random variables X and Y are independent.

- (e) What is $p_{X,Y}(2, 2)$?
 (If there is not enough information to determine the answer, say so.)
- (f) What is $p_{X,Y|B}(2, 2 | B)$?
 (If there is not enough information to determine the answer, say so.)

3. **Problem 2.33, page 128 in the text.** A coin that has probability of heads equal to p is tossed successively and independently until a head comes twice in a row or a tail comes twice in a row. Find the expected value of the number of tosses.

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